United States Patent Application for Evaluation
D'Zmura

Invention in Finance

Inventor: David Andrew D'Zmura
Applicant: David Andrew D'Zmura

Section I

BACKGROUND OF THE SECTION

The first aspect of this invention relates to the methods and processes for analytic valuation of financial instruments and estimation of their pricing sensitivity, that is, their characterizable traits and values which determine the instrument's changes in price. The methods presented herein are endogenous, that is, the technology relies upon the individual security itself, rather than upon the approximation of an individual security given a market-determined modelling process. For example, in the case of U.S. Treasury instruments, each Treasury has the primary characteristics of maturity, coupon rate, and yield (indicative of price), while being devoid of credit risk and special repayment ("optionality") features such as early call provisions. The aggregate of Treasury instruments, over the maturity term spectrum, comprises the "yield curve", this being an external indicator of value for Treasuries, providing an external basis for determining propensities of an individual Treasury to change in price for a change in the yield curve. This invention, however, uses the individual security itself to determine its propensity to change in price, and not an external relational method. Because of this, the invention provides an analytic means counter to the prevailing standard, and hence, provides the ability to contrast and compare the results determined through standard analytic practice.

Within the investment and trading community, great interest is placed on analytic means to value a security, to determine its sensitivities and to determine its propensity for price change. First, sound analytic means are used to determine how any given instrument compares with other instruments of the same security class, or other classes, to reveal which might be a preferable investment choice. Second, analytic methods are crucial to asset/liability matching which is core to insurance company and pension fund management, since the operators need to meet future financial liabilities with investment choices which are made in the present. Third, analytic methods are key to risk management, wherein a security must be analysed correctly if hedging is to operate properly. And fourth, analytic means are used to determine the relative value between comparable securities or between a security and its related market, revealing premiums or discounts in pricing as well as determining the oppportunity to arbitrage between instruments or between an instrument and a market.

The second aspect of the invention relates to the methods and process necessary to compose a new typus of financial instrument which replicates the characteristics of a specified security, or group thereof. This process and product are of significant value within the financial community for two prime reasons. First, by creating replicated securities, the palette of investment choices is greatly expanded, giving investors increased opportunities to find securities with the exact characterisitics which they seek. For instance, in U.S. Treasuries, there are only about 220 different securities which can be purchased over the entire thirty year spectrum. Consequently, from the point-of-view of an asset/liability manager, for example, the manager must select from Treasuries which may not match his or her requirements for maturity or duration exactly, therefore introducing risk to the portfolio downstream.

The manufacturing process of replicants detailed herein allows for automatic generation of replicated securities, yet also for the creation of replicants according to customer-specified requirements, and hence, replicants may provide exactly the conditions which are sought. Moreover, because replicants are equivalent for a specified characteristic, for instance, cash-flows, they show small variances in their other characteristics from the security being replicated.

This is the second primary benefit of replicants. These variations can opportune advantageous distinctions for any type of investor or portfolio manager, such as a cheaper purchase price, desirable improvements in price change propensity and the maturity. Also, because a replicant security is equivalent to a targeted security in some fundamental determinant, for instance cash flows, yet slightly differentiable, eg. slightly different pricing, occassions can arise that the replicant and the target offer an opportunity for arbitrage profit.

The third aspect of this invention relates to the automated execution of profitable, arbitrage trading positions which are identified by the analytic and replication methods. For instance, the analytic valuation methods can reveal when a security, or portfolio, is trading at a discount or premium to other directly comparable securities. Because of the fungible relation between securities of the same type and characteristics, such as Treasuries with duration of two years, one can expect the market to price them the same, given their equality. When the pricing of one similar instrument moves out of line with another, astute arbitrageurs buy the cheaper instrument and sell the more expensive one, knowing that market efficiencies will return the pricing of these two instruments to an equal basis.

This action on the part of arbitrageurs is the force which ensures this convergence between equal securities. Yet, because these differentials are often momentary or fleeting, an automated arbitrage engine is essential to capture these opportunities. Thus, this third aspect of the invention is designed to render profitable trading to the user, while providing efficiency within the market for the pricing of traded securities. As this invention centers upon the analytic valuation methods and the manufacture of replicant securities, so is its arbitrage engine based upon transactions using these means.

THE PRIOR ART

This section discusses previous and presently existent methods and processes which are relevant to the analytic valuation technology and the manufacture of replicant securities that are core to this invention.

To begin this section, consider the standard textbook form which relates the change in price of a bond to endogenous factors, specifically, the bond's yield-to-maturity (YTM), duration and convexity, with these values calculated from a bond's price, maturity and coupon. This formulation is, among other sources, found in Fabozzi, Frank J. *Bond Markets, Analysis and Strategies*. Third Edition. Prentice Hall. New Jersey. 1996. pp 71-2.

The generalized form for estimating a change in price, relational to duration and convexity, is estimated according to a change in the yield-to-maturity (YTM) of the bond, and is given as (called herein, the "duration/convexity factorization"): Formula S.1:

A) Δ Price, due to Duration = (-Duration) × Δ YTM

B) Δ Price, due to Convexity = 1/2 × Convexity × (Δ YTM)^2

C) Δ Price, due to Duration & Convexity = A) + B):

where the values for Duration and Convexity are those at the beginning of the timeframe.

The standard formula calculating YTM from price, maturity and coupon is (Tuckman, Bruce. *Fixed Income Securities*. Wiley. New York. 1995. p. 32.): Formula S.2:

$$\text{Price} = \frac{C}{2} \sum_{T=1}^{2T} (1 + Y/2)^{-T} + (1 + Y/2)^{-2T}$$

where C=Coupon Y=YTM T=Maturity in Years.

Duration (modified annualized) is formulated (Fabozzi, ed. inventor): Formula S.3:

$$\text{Durmodan} = \frac{\dfrac{C}{Y^2}\left[1 - \dfrac{1}{(1+Y)^T}\right] + \dfrac{T(100 - C/Y)}{(1 + Y)^{T+1}}}{2P}$$

where D = $\Delta P/\Delta YTM$
Y = YTM
T = (Maturity x 2)
C = Coupon
P = Price

Convexity (annualized, per $100 par) is conventionally determined, like the general forms of duration, through Taylor series approximations, resulting in the following form (Fabozzi, ed. inventor): Formula S.4:

Convexity (annualized per $100 par) =

$$\frac{\dfrac{2C}{Y^3}\left[1 - \dfrac{1}{(1+Y)^T}\right] - \dfrac{2CT}{Y^2(1 + Y)^{T+2}} + \dfrac{T(T+1)(100 - C/Y)}{(1 + Y)^{T+2}}}{4P}$$

where
Y = YTM
T = (Maturity x 2)
C = Coupon
P = Price

There are a number of major problems with this methodology in its standard form. First, while the method captures the direction of price movement with respect to a change in YTM, its accuracy in estimating the magnitude of change is rather poor, as shown by the next section's tests (accuracy range of 55% to 84%), and hence, it cannot be used for any demanding application, such as to construct hedge ratios, to tune immunization strategies, or to compare replications with varying sensitivities.

Second, and of perhaps greater import, this formulation is compromised by the definitional relationship between price and YTM, whereby YTM is merely another way to quote a bond's price. Hence, at best, were this formulation to relate a change in price to a change in YTM accurately, then, by its tautological nature, it would indicate that it factors duration and convexity equivalent to the first derivative of the price/YTM equation.

In other words, since the standard calculations for duration and convexity use YTM and price as inputs, these two terms being dependent and interrelated, and since the duration and convexity values are multiplied by the change in YTM for the duration/convexity factorization, all values should allign by definitional necessity. But this is not the case, indicating that this method, and its assumptions, are critically flawed.

The first derivative of the YTM calculation, modified annualized duration, is conventionally assumed to show the change in price accompanying a change in YTM, yet, given its definitional construction using both price and YTM, it is a compromised indicator. Note that modified annualized duration contains four variables: coupon, yield, maturity and price, and consider the unwieldy, cloudy equation which duration becomes once you substitute the Price/YTM formula into the denominator, in the place of (2 x Price).

The final part of this discussion now turns to an examination of convexity. The usual assumption for convexity is that it is equivalent to the second derivative of the price/YTM formula, and indicates the change in curvature of the first derivative. In other words, convexity reveals how the bond's propensity to change in price is transforming.

The typical formulation of convexity is based upon the duration expression, and consequently, is again rendered mathematically from the four variables of coupon, maturity, yield-to-maturity and price. Moreover, the standard form of convexity is held to represent the change in the change in price caused by a change in YTM. This derivation can be criticized along lines similar to those raised previously regarding duration.

In conclusion to the prior endogenous analytical art, we have noted that the method is both inaccurate as well as mired in a tautological confusion. This process never separates price from YTM, and estimates a bond's prospective change in price using intermediate steps which at best should render a definitional relation. Unfortunately, the method does not do this, although this method continues to be preached and practiced.

Accordingly, modern precision fixed-income professionals should not use this endogenous method to estimate how the price of a bond changes. Rather, many estimate the change in a bond's price by using the exogenous calibrations afforded by OAS models and binomial interest rate trees. Yield curves, or more generally, the term structure of interest rates, are posed as changed, and by this exogenous means which relates a variety of rate types, the value of the bond is recalculated.

In the absence of a sound endogenous methodology, the fixed-income professional is limited. A primary constraint is that the yield curve cannot be independently assessed as to mispricings within its composition. Additionally, because the exogenous methods rely on fungible valuations, such as the liklihood of upward versus downward branching within the tree, or the establishment of the volatility level within interest rates or the bond under evaluation, the lack of an independent, external method for comparison hampers the vital critique of these subjectively-, probabalistically-, or historically-based inputs.

Regretably, the critical interrelationships of rates, being the par yield, zero yield and forward yield curves, which underlie exogenous analytic valuation methodologies are not as robust as would be hoped if these methods were to be unassailable. Particularly fundamental is its presumption that, for a vanilla bond trading at par, its yield will be equal to the zero spot yield of the same maturity.

Yet, the vanilla portfolio presented herein for testing, trading at an average value virtually of par, contraindicates this assertion, since its average yield widely diverges from the zero spot yield for the maturity corresponding to the portfolio's average maturity. This breakdown in method for only a simple portfolio undermines the credence of rigorous reliance upon exogenous methods.

Moreover, the conventional conceptualization regarding forward rates, as relating the future path of interest rates, is contradicted by both empirical study and common sense: future yields are more correlated to current yields than to forward rates; if forwards indicated future rates, than the economy would be continually subjected to rising rates. Rather, forwards are simply an option imbedded in the yield structure, commanding a premium for the certainty provided by locking in a rate for a future date today.

Thus, the discovery of an endogenous methodology for characterizing a bond's endogenous aptitude to change in price, not subject to tautological definition, is of genuine value to the securities professional. It is to this end, that the technologies presented in the next section have been developed.

As pertains to the second key aspect of this invention, the manufacture of "replicated equivalent primary securities", the prior art does not directly cover this invention, but the universe of securities and financial instruments is briefly explored here. A survey of leading academic texts discloses in-depth discussion of currently marketed and traded financial instruments. The first category of objects are the primary securities, being Treasuries, Bonds and Stocks. These are financial objects, separate and individual, being the direct physicality of ownership and debt entitlements. These financial objects are stores of value themselves, and do not depend upon other assets for their valuation.

In addition to this primary category of financial instruments, there are several others receiving great commercial and academic interest. However, because they are generally derivatives and/or derivative combinations, they are therefore not composed from the proper object of matter (Treasuries, bonds, and stocks) in finance, since they are removed from direct ownership and indenture. A "derivative" is a financial asset whose value depends upon some other, removed (underlying) asset. These instruments include (see Gastineau, *Dictionary of Financial Risk Management*):

- derivatives: these are not physical, actual or severable Treasuries, bonds, or stocks, but rather an option, forward, future, swap, warrant or index thereon;

- synthetic securities: these instruments combine selected instruments from the various categories (ie. a future and a stock in combination) to synthesize a specific security;

- replicating portfolios: combinations of market securities, cash, and borrowing that reproduce the return pattern of an option or option-based instrument;

- linked notes: a recombination of a fixed-income security's interest and principal payment obligations, ie. reforming the original note from previously separated coupon and principal components.

Thus, the universe of financial instruments is essentially comprised of two classes: the primary objects (ie. bonds, stocks) and the derivatives (ie. options, forwards, futures etc), plus mixtures of these two categories (ie. synthetic securities, replicating portfolios etc.). Hence, the prior art of marketed financial instruments is limited to securities, derivatives and hybrids of these types.

To the extent that the prior art does not address the analytic methods nor the replicated equivalent primary securities, th prior art cannot and does not exist as pertains to automated trading and arbitrage as based upon these technologies and instruments.

DETAILED DESCRIPTION OF INVENTION

This section lays out the invention, and demonstrates its operation with examples. The examples also serve to show how the prior art in endogenous analytics fares when facing the same data. For the purposes of the example implementation, a portfolio of U.S. Treasury instruments is used, and the pricing for this portfolio is tracked over a period. As concerns the generation of "replicated equivalent primary securities", examples which replicate given Treasury instruments are shown.

The choice of Treasury instruments does not limit these inventions to Treasury instruments, rather, these instruments have been chosen because they have the most transparent interrelations, and hence, serve as a fundamental and incontravertible test for the inventions' efficacy and superiority over the prior art.

This section utilizes the data from this financial portfolio to demonstrate operation of the analytic engine. This data is fed as arrays into the processing engine. This portfolio is considered in aggregate, that is, it is posed as having a composite value for all manner of the common bond values, such as face value, price, accrued interest, duration, etc.

This aggregation is permissable since all issues share the same classification characteristics, being Treasury notes. Moreover, this aggregation allows finer scaling of valuation, such as pricing, which otherwise could only ratchet incrementally by 1/32nds on an issue-by-issue basis.

<u>Summary: Test Portfolio of U.S. Treasuries, 3/22/96-4/25/96</u>

Issue	1)	2)
Maturity	11/96	5/97
Coupon	4.3875%	6.125%
Matur, yrs fr 3/22	.647541	1.14481
Matur, yrs fr 4/3	.614754	1.11475
Matur, yrs fr. 4/25	.505464	1.05464
Ask Yield, 3/22	5.23%	5.58%
Ask Yield, 4/3	5.34%	5.53%
Ask Yield, 4/25	5.26%	5.59%
Price 3/22	99:12	100:19
Price 4/3	99:13	100:19
Price 4/25	99:14	100:16
Face Value	$70,000,000	$100,000,000
AI, 3/22	$1,082,490	- 0-
AI, 4/3	$1,193,186	$217,555
AI, 4/25	$1,367,797	$585,724
Full Value 3/22	$70,644,990	$100,593,750
Full Value 4/33	$70,767,561	$100,811,305
Full Value 4/25	$70,974,047	$101,085,724

Issue	3)	4)	5)
Maturity	10/97	8/98	3/99
Coupon	5.75%	5.875%	5.875%
Matur, yrs fr 3/22	1.56438	2.40274	2.98082
Matur, yrs fr 4/3	1.53160	2.36995	2.94804
Matur, yrs fr 4/25	1.46995	2.30601	2.88524
Ask Yield 3/22	5.60%	5.79%	5.87%
Ask Yield 4/3	5.63%	5.85%	5.90%
Ask Yield 4/25	5.75%	5.98%	6.07%
Price 3/22	100:03	100:04	99:30
Price 4/3	100:01	100:00	99:28
Price 4/25	99:28	99:20	99:11
Face Value	$40,000,000	$120,000,000	$40,000,000
AI, 3/22	$999,180	$693,443	$44,945
AI, 4/3	$1,074,590	$924,590	$121,995
AI, 4/25	$1,219,126	$1,367,623	$269,672
Full Value 3/22	$41,036,680	$120,843,443	$40,019,945
Full Value 4/3	$41,012,090	$120,924,590	$40,071,995
Full Value 4/25	$41,169,126	$120,917,623	$40,007,172

Issue	6)	7)
Maturity	6/00	2/01
Coupon	5.875%	5.625%
Matur, yrs fr 3/22	4.23288	4.90274
Matur, yrs fr 4/3	4.20009	4.86995
Matur, yrs fr 4/25	4.13661	4.80601
Ask Yield 3/22	6.04%	6.03%
Ask Yield 4/3	6.04%	6.04%
Ask Yield 4/25	6.25%	6.28%
Price 3/22	99:10	98:07
Price 4/3	99:11	98:07
Price 4/25	98:16	97:05
Face Value	$80,000,000	$60,000,000
AI, 3/22	$1,258,470	$331,967
AI, 4/3	$1,412,568	$442,623
AI, 4/25	$1,707,923	$654,713
Full Value 3/22	$80,708,470	$59,263,217
Full Value 4/3	$80,887,568	$59,373,873
Full Value 4/25	$80,507,923	$58,948,463

All relevant values for the portfolio are on an issue-by-issue basis, and are shown for the dates of 3/22, 4/3 and 4/25/96. The portfolio has a face value of $510,000,000. The analytic engine calculates aggregate values since it is for a portfolio, these values are shown in the table that follows, with these values then moved into further processing. Were this a single instrument, then the values would be simply that of the single instrument. Were this for an institution's global exposures, then the comprehensive exposures would be performed in the aggregated portfolio manner.

Aggregate Value Calculations for Portfolio

Accrued Interest = $\sum$ AI, for all issues
Present Value = $\sum$ (AI + (Best Bid Price $\times$ Face Value)), for all issues
Implied Price = (Present Value - Accrued Interest)/\$510,000,000.

Portfolio Coefficient, per date = Present Value, per issue/Portfolio Present Value
Coupon (Portf.) = $\sum$ Coupon $\times$ Portfolio Coefficient, for all issues
Maturity (Portf.) (in years) = $\sum$ Maturity $\times$ Portfolio Coefficient, for all issues
YTM (Portf.) = $\sum$ YTM $\times$ Portfolio Coefficient, for all issues
Duration (Portf.) (in years) = $\sum$ Duration $\times$ Portfolio Coefficient, for all issues
Convexity (Portf.) = $\sum$ Convexity $\times$ Portfolio Coefficient, for all issues.

Aggregate Values for Portfolio

Test Date	3/22	4/3	4/25
Accrued Interest	\$4,749,907	\$5,387,107	\$7,172,578
Present Value	\$513,449,907	\$513,848,982	\$513,610,078
Implied Price	.99745098	.99698407	.99301471
Portfolio Coefficient			
11/96	.1375888	.1377205	.138186
5/97	.1959174	.1961886	.196814
10/97	.0799234	.0798135	.077917
8/98	.2353559	.235331	.235427
3/99	.0779432	.077984	.077894
6/00	.1572929	.157415	.156749
2/01	.1159784	.115547	.114773
Coupon (Portf.)	5.680331%	5.680322%	5.667059%
Maturity (Portf.)	2.470660	2.437096	2.359601
YTM (Portf.)	5.730002%	5.755183%	5.856197%
Duration (Portf.)	2.222031	2.191867	2.130696
Convexity (Portf.)	7.847886	7.695562	7.389558

To understand the next step of the new endogenous analytic methodological process, consider the theoretical axiom central to the process:

for any security class, there is a function which relates its market price (P) to a limited set of endogenous variables for the security, such that within and across any class of securities, two or more securities can be compared: Formula 1.1:

$P = f\ \{C, Y, T\}$ where C, Y, and T are variables ndogenous to the security

C = Cash Flows, relating periodicity, certainty, value changes to principal
Y = Yield, a single term relating security's return, relative to P, C, T
T = Time, a terminal or contiuous measure of the life of the security.

The single term, yield, is the next object found in the processing: that single level of return relating the pricing of a security to its endogenous variables. This rate has never been established, in convincing manner or method, in literature or practice, for even the simplest class of securities, being U.S. Treasury instruments. For in Treasuries, the other variables are simple in composition (C, being coupon; T, being years to maturity), and together with the Treasury's price, are known with certainty

Were such a yield term identified, then, corresponding to its reference to market rates of return, it must show relativity to that appropriate market's yield structure. In the case of U.S. Treasuries, this market yield is the zero spot rate of that same maturity, and given the transparency of Treasuries, it should therefore fairly equal that appropriate zero spot yield. Moreover, should such a yield be identified, then, to be fully functional and useful, it must anchor some factorization process which accurately states the change in the price of the security given a change in this singular yield (called hereafter, the "governing yield").

The "governing yield" is not the (Treasury) portfolio's YTM. In the table that follows, this is indeed confirmed. Within this same output table, the yield posed as the governing yield is calculated for the portfolio. The derivation of this yield (for the portfolio) is computed by weighing each issue within the portfolio by a measure of its carrying time, ie. maturity, and then dividing the summation of all weighted issues by the average time value of the portfolio, ie. average portfolio maturity.

This yield, termed "Yield M" herein, respective of the use of maturity values, is calculated as follows: Formula 1.2

$$\text{Yield M} = \frac{\sum \text{(Maturity} \times \text{Portfolio Coefficient} \times \text{YTM), for all issues}}{\sum \text{(Maturity} \times \text{Portfolio Coefficient), for all issues}}$$

For a single Treasury, Yield M is is calculated letting the summation be for all cash flows over the life of the instrument. To compare coupon-bearing instruments with zero-coupon instruments, and in particular, to relate a coupon-bearing instrument's governing yield to the market's yield, when this latter yield is defined by zero-coupon instruments, Yield M is modified by multiplying the above formula by $(1 - \text{Coupon}^2)$, where coupon is expressed as a decimal (this adjusted yield is termed herein, "Yield Md": Formula 1.2a).

For comparison testing, the Treasury market's zero spot rate for the future date corresponding to the portfolio's maturity is reported and compared to the yields generated for each test date.

Test Date		3/22	4/3	4/25
Maturity (Portf.)		2.462223	2.436861	2.353077
Maturity (Future Date)		9/10/98	9/10/98	9/5/98
Zero Spot	8/98	5.83%	5.86%	6.04%
	11/98	5.86%	5.90%	6.09%
linear	9/98	5.84%	5.87%	6.06%
fitted	9/10/98	5.845%	5.875%	6.065%
Yield M (Portf.)		**5.87129004%**	**5.89269332%**	**6.0661141%**
Yield Md		**5.8523%**	**5.8737%**	**6.047%**
YTM (Portf.)		5.73000157%	5.75518286%	5.8561971%

From this table, one sees that Yield M closely mirrors the actual zero spot rat corresponding to the portfolio's maturity on each of the three test dates, moving from values roughly 3 b.p. higher than the zero spot at the initial period, to values basically identical to the zero spot at the third period. Yield Md adjusts Yield M to the market's zero spot yield, and by the third period dips beneath it. In contrast, YTM shows that it has little clear relation to market yields of similar maturity by its poor tracking.

This convergent movement reflects the reversional characteristic of the governing yield to the market spot yield, whereby the security's governing yield remains in close proximity to the actual spot, due to market trading efficiencies, revolving about the spot yield. Yield Md essentially adjusts Yield M to the market yield, and by dipping beneath the spot yield, suggests a reversional tension is building to readjust in the opposite direction.

The reversional nature is not unexpected, given both the fungibility of Treasury instruments amongst themselves and the origin of the zero spot yield from the pricing of Treasuries along the yield curve. Such a reversional characteristic also indicates trading opportunities to capture the premium or discount spread of portfolios and instruments vis-a-vis the zero spot yield.

Since Yield M defines the governing yield for the portfolio with a good degree of accuracy, the question arises as to whether the change in this yieldcan be used to approximate the resultant change in price. This should be the case, if this yield indeed represent the proper yield value. The next steps of the analytic processing show the method to derive this pricing sensitivity, as well as confirming its accuracy.

First, the formula which utilizes standard duration and convexity values to approximate the change in price accompanying a change in the yield is applied to the data. To contrast the accuracy of Yield M, YTM (the basis of the prior art) is also used.

The table which follows shows the result: the governing yield, not YTM, is the yield which underlies the duration/convexity factorization. While the use of ΔYTM shows itself to be an inaccurate tool, ΔYield M repeatedly estimates the resultant change in price to a high degree of accuracy, posting accuracy of 96%- 98% for each test period.

Beyond implying that duration and convexity may fully factor the sensitivity of a bond to change in price for a given change in governing yield, it is further suggested that the calculation of the governing yield, coupled with this factorization, affords the practitioner an endogenous means to convey and to critique the market's term structure. Unlike exogenous methods, its calculation is direct and simple by comparison. Moreover, Yield M posts a degree of accuracy required for precision applications.

Test Period	3/22 - 4/3	4/3 - 4/25	3/22 - 4/25
Actual Δ Yield M	0.0002140328	0.0017342077	0.001948241
Actual Δ YTM	0.0002516720	0.0010101424	0.001261814
Duration (Portf.)	2.222031	2.191867	2.222031
Convexity (Portf.)	7.847886	7.695562	7.847886
Computing Duration, Convexity Factorization:			
Estimated Δ Price, M	**-0.0004754078**	**-0.00378958**	**-0.004314158**
Estimated Δ Price, YTM	**-0.0005589743**	**-0.00221017**	**-0.002796069**
Actual Δ Price	**-0.000466911**	**-0.003969363**	**-0.004436274**
% Accuracy Yield M	**98.2 %**	**95.5 %**	**97.2 %**
% Accuracy YTM	83.5 %	55.7 %	63.1 %

A third key improvement of the invention lies in its method of computing modified duration, the first derivative of the YTM formula, which represents a change in price for a change in yield. Recall that the standard formula relating YTM and price is (Tuckman, Bruce. *Fixed Income Securities*. Wiley. New York. 1995. p. 32.):

$$\text{Price} = \frac{C}{2}\sum_{T=1}^{2T} (1 + Y/2)^{-T} + (1 + Y/2)^{-2T}$$

where C=Coupon Y=YTM T=Maturity in Years.

This formula can be rewritten, without a summation form, as

$$\text{Price} = \frac{C}{Y}\left(1 - (1 + Y/2)^{-2T}\right) + (1 + Y/2)^{-2T}$$

From this latter form, the invention uses the chain rule of differential calculus to formulate the first derivative (the inventor): Formula 1.3:

$$\frac{\partial P}{\partial Y} = K = \frac{-C}{Y^2}\left(1 - (1 + Y/2)^{-2T}\right) + \frac{C}{Y}(T + TY/2)^{-2T-1} - (T + TY/2)^{-2T-1}$$

where C=Coupon (decimal entry, portfolio) Y=YTM (portfolio) T=Maturity in Years (portfolio) $\partial Y=\Delta$Rate $\partial P=\Delta$Price. (with $\partial Y/\partial P$ = "K")

Since this derivative is held, in the prior art, as mapping the change in price accompanying a change in YTM, and given the definitional relationship between price and YTM, this would remain a questionable indicator. Yet, as demonstrated in the output table above, the derivative actually shows change in price for a change in the governing yield.

Moreover, because the derivative is computed solely from the internal values (C, Y, and T) of the instrument under consideration (in this case, the Treasury portfolio), without simulataneously invoking both price and yield, it provides a true endogenous means to estimate the change in price given a change in the rate, not subject to tautological definitionalism.

Because the formula calculates a coefficient to express $\partial P/\partial Y$, this coefficient can be applied in independent contexts, allowing diverse bonds or portfolios to be compared (though with reference to their appropriate date on the term structure). This coefficient, $\partial P/\partial Y$, is renamed, K, to distinguish it from standard means of derivation within th analytic business logic.

Compare the formulation of K with that of duration (modified annualized):

$$\frac{\partial P}{\partial Y} = K = \frac{-C}{Y^2}\left(1 - (1 + Y/2)^{-2T}\right) + \frac{C}{Y}(T + TY/2)^{-2T-1} - (T + TY/2)^{-2T-1}$$

where C=Coupon (decimal entry, portfolio) Y=YTM (portfolio) T=Maturity in Years (portfolio) $\partial Y=\Delta$Yield M $\partial P=\Delta$Price. (with $\partial P/\partial Y$ = K)

K can thus be rewritten as: Formula 1.3:

$$\frac{\partial P}{\partial Y} = K = \frac{-C}{Y^2} + \frac{C}{Y^2}(1 + Y/2)^{-2T} - (1 - C/Y)(T + TY/2)^{-2T-1}$$

whereas duration (modified annualized) is formulated (Fabozzi, ed. inventor): S.3:

$$\text{Durmodan} = \frac{\dfrac{C}{Y^2}\left[1 - \dfrac{1}{(1+Y)^T}\right] + \dfrac{T(100 - C/Y)}{(1 + Y)^{T+1}}}{2P}$$

where D = $\Delta P/\Delta YTM$
Y = YTM
T = (Maturity x 2)
C = Coupon
P = Price

First, note that Durmodan contains four variables: coupon, yield, maturity and price, whereas K contains only coupon, yield and maturity. This is a marked improvement, since price and yield, being variables which are definitionally related, should not be used together if concerns of tautology and analytic clarity are to be countenanced. To see this better, consider the unwieldy, cloudy equation which duration becomes once you substitute the Price/YTM formula into the denominator of duration, in the place of (2 x Price).

Second, observe that K is of inverse sign to the conventional duration, reflecting the fact that as the yield rises, the price declines. Hence, K clearly depicts the primary pricing sensitivity of the security. K is derived expressly for its single purpose, and ideas of K representing time weighted mass is a secondary manipulation of its meaning.

Third, observe that K states this sensitivity of change in price with respect to a change in the governing yield, rather than to a change in YTM, which is the method of conventional duration logic. It is mentioned that the conventional duration calculation can be used with fairly good results (as has been done to this point in the factorization numbers) assuming the change in governing yield is used rather than a change in YTM. But regardless, it is critical to realize that K estimates a pricing relation for a change in the yield, and that, preceding the use of K analytically, a value for the governing yield must be derived so as to be able to apply K in the proper context.

Fourth, observe that K does not equal the inverse of duration, as shown in the table that follows. This results from mathematical discrepancies between the formulation of values for K and duration (modified annualized). Part of this stems from the fact that the duration calculation removes the halving process of Y (in its discounting function) which is found in the formulation of K. Coupled with the addition of one within both renditions of the exponent, the rate of discounts differ.

The formulas also diverge in the second part of the expression, with regards to the relativity of T. For values of K, T is distributed through the discounting denominator, while for duration, T resides in the numerator. As a consequence, K incorporates T as a factor of the discounting, while duration holds T as a factor to be discounted.

Lastly, K is an elegant, exact mathematical form, being the derivative as found by using the chain rule of differential calculus. In contrast, the standard formulation of modified duration is a Taylor approximation, which is a summation of numerous partial derivatives, and whose accuracy only approaches the true solution when this number of partials approaches infinity. Thus, the use of Formula 1.3 is preferable to the use of S.3, even if all conventional variables are used to calculate duration (ie. using YTM instead of Yield M), and this permutation is called Formula 1.3a.

K vs. Durmodan	3/22	4/3	4/25
Coupon (Portf.)	5.680330985%	5.680322119%	5.66705895%
Maturity (Portf.)	2.4706604	2.437096	2.359601
YTM (Portf.)	5.73000157%	5.75518286%	5.8561971%
Price (Portf.)	99.745098	99.698407	99.301471
(Price N/A for K)			
K	-2.25389446	-2.21483844	-2.10426651
Durmodan	2.09611877	2.07102626	2.01633865

Returning now to the "duration"/convexity factorization, the next output of the analytic engine is generated using the K value in place of the duration value, as appropriate to each test period, recalculating the estimated change in price stemming from the change in the governing yield. Given that K expresses itself as a negative number, the negation of duration (i.e. -Duration x ΔY) is eliminated from the factorization. The computational code within the analytic engine is altered accordingly (ie. Duration x ΔY, where Duration = K, and ΔY = ∂Y, or ΔYield M). The use of ΔYield M is the empirical value, whereas ∂Y is a "theoretical" value, this being the value used in this particular table.

Test Period	3/22-4/3	4/3-4/25	3/22-4/25
K	-2.25389446	-2.21483844	-2.25389446
Convexity (Portf.)	7.847886	7.695562	7.847886
∂Y	0.0002071580	0.0017921768	0.001968276
Estimated ΔP	-0.000466744	-0.003957023	-0.004421085
Actual ΔP	-0.000466911	-0.003969363	-0.004436274
Accuracy %	**99.96%**	**99.69%**	**99.66%**
Error %	**0.04%**	**0.31%**	**0.34%**

Clearly, the usage of K, in the place of duration, shows substantial improvement, shaving the error further from a range of 1.3 - 1.8% to just 0.04 - 0.34%, a reduction of error by four to forty times. Overall, the accuracy of the methodology has reached above the top percentile, indicating that the processing is precisely on target, indeed has approached a perfected solution.

In fact, relative to the conventional factorization using ΔYTM and duration which registered an accuracy of just 55 - 84% (error of 16 - 45%), this methodology has reduced error a remarkable 50 to 1000 times. Hence, this integrated methodological process avails the practioner a level of great accuracy, quantifying the mechanics of future price movements to current endogenous values, and thus, it can be used in the most demanding of applications where precise quantification of price sensitivity is required.

The final part of this analytic processing now turns to an examination of convexity. Under a useful assumption for convexity, this is equivalent to the second derivative of the price/YTM formula, and indicates the change in curvature of the first derivative. In other words, convexity reveals how the bond's propensity to change in price, as stemming from a change in the governing yield, is transforming.

The typical formulation of convexity is based upon the duration expression, and consequently, is rendered mathematically from the four variables of coupon, maturity, yield rate and price. Moreover, the standard form of convexity is held to represent the change in the change in price caused by a change in YTM. This derivation can be criticized along lines similar to those raised previously regarding duration.

Convexity (annualized, per $100 par) is conventionally determined, like the general forms of duration, through Taylor series approximations, resulting in the following form (Fabozzi, ed. inventor): S.4:

$$\text{Convexity (annualized per \$100 par)} = \frac{\dfrac{2C}{Y^3}\left[1 - \dfrac{1}{(1+Y)^T}\right] - \dfrac{2CT}{Y^2(1+Y)^{T+2}} + \dfrac{T(T+1)(100 - C/Y)}{(1+Y)^{T+2}}}{4P}$$

where
Y = YTM
T = (Maturity x 2)
C = Coupon
P = Price

In place of this formulation of convexity, another is substituted being the derivative of the expression for K. Again, like the derivation of K, this formulation is an exact measure, being found through the chain rule of differential calculus. To accompany its usage herein, let this formula be termed "V": Formula 1.4:

$$V = \frac{2C}{Y^3} - \frac{\dfrac{2C}{Y^3}}{(1 + Y/2)^{2T}} - \frac{\dfrac{CT}{Y^2}}{(1 + Y/2)^{2T+1}} - \frac{\dfrac{C}{Y^2}}{(T + TY/2)^{2T+1}} + \frac{(1 + C/Y)(T^2 + T/2)}{(T + TY/2)^{2T+2}}$$

where
C=Coupon (decimal, portfolio) T=Maturity (years, portfolio)
Y=(Yield M - YTM), both values for the portfolio, expressed in basis points
V= ΔP with respect to the differential between the governing yield and market spot

The differences in formulation between V and standard convexity are apparent visually, and procede along the same lines as the previous commentary concerning K and duration. However, of particular note is that the calculation of V requires the estimation of Yield M, the governing yield, and that the value, Y, is actually the difference between the governing yield for the portfolio and its YTM. Nontheless, as with K, the standard input of YTM can be used herein as well with good results (Formula 1.4a), since V is an xact derivative rather than a Taylor approximation.

Principally, however the coefficient V is not an expression of the change in the rate of change in price for a given change in yield, but rather, it defines the reversional tension which exists between the internal, governing, yield of the portfolio and the market spot.

Hence, as would be expected under this assertion, the value for V will be higher when the differential between the internal and external values is substantial, whereas V will be smaller when the differential is negligible.

The coefficient V, or convexity as understood in this terminology, is the factor which moves the governing yield of the portfolio into allignment with the market's spot rate. This assertion is supported by the following table, which shows not only that the calculation of V renders a convexity value distinct from the conventional convexity, but also is reflective of this underlying spread dynamic in the anticipated manner. As can be also seen therein, the conventional convexity does not capture this nuance.

<u>V vs. Convexity</u>	3/22	4/3	4/25
Rate M	5.87129004%	5.89269332%	6.0661141%
YTM	5.73000157%	5.75518286%	5.8561971%
Rate M - YTM (b.p.)	0.14128852	0.13751046	0.2099176
Coupon	5.680330985%	5.680322117%	5.66705895%
Maturity	2.4706604	2.437096	2.359601
Price (N/A for V)	99.745098	99.698407	99.301471
V	**6.84893917**	**7.14436415**	**2.89621154**
Convexity	6.05221587	5.91149933	5.60084222
Market Spot Yield	5.845%	5.875%	6.065%
Rate M - Zero Spot	**0.026%**	**0.018%**	**0.001%**

Similarly, we might expect that the "duration/convexity" factorization, making use of both the K and V values, could post the best accuracy of all measures taken within this section. Indeed, the use of K and V does improve the strong results demonstrated by the use of K alone. This improvement, reducing error further by about one-quarter to a meager 0.03 - 0.3%, is summarized below.

<u>Test Period</u>	3/22-4/3	4/3-4/25	3/22-4/25
∂Y	0.000207158	0.0017921768	0.001968276
K	-2.25389446	-2.21483844	-2.25389446
V	6.80893917	7.14436415	2.89621154
Estimated ΔP	-0.000466766	-0.003957909	-0.004423097
Actual ΔP	-0.000466911	-0.003969363	-0.004436274
Accuracy %	**99.97%**	**99.71%**	**99.70%**
Error %	0.03%	0.29%	0.30%

Per th formulations and calculations of the governing yield, K and V, the factorization previously termed the "duration/convexity factorization", which is to estimate the change in price to the portfolio given a change in the yield, can be restated as: Formula 1.5:

$$\text{Estimated } \Delta P = (K \times \partial Y) + (1/2 \times V \times (\partial Y)^2)$$

where

$\partial Y = \Delta P/K$ or ∂Y may be approximated as ΔYield M (Formula 1.2

K = Formula 1.3 and V = Formula 1.4.

In conclusion, this section has demonstrated that a proper yield value does underlie the pricing of a Treasury portfolio. This yield can be easily approximated in value through the endogenous characteristics of C, Y, and T. Owing to its importance, this yield has been termed the "governing yield", and, though tracking near to the market's spot yield for corresponding maturity, it is distinguishable therefrom. Further, it appears that the governing yield has reversional tendencies to the zero spot, which is a function of the market efficiencies and fungibility of securities of the same typus.

It has also been demonstrated that the governing yield is the yield of reference within the duration/convexity factorization, which estimates a change in the price of the portfolio given a change in the governing yield. Once this distinction from the traditional understanding of the factorization is countenanced, the accuracy of the estimations greatly improves, reaching into and above the top percentile.

Part of the improvement in factorization accuracy stems from the mathematical derivations of duration and convexity which are presented herein. These formulations are precise, elegant mathematical formulae, endogenous in origin, but without the tauological coexistence of price and yield values which mar prior calculations. Also, duration and convexity are reinterpreted as to their meaning and context of application, demonstrating a rigorous usefulness and cohesive intelligibility superior to that of their conventional derivation and interpretation.

Method, .. Process and Manufacture of Replicated Equivalent Primary Securities

By replication, it is herein meant that a targeted primary security, such as a Treasury, Bond or Stock, is reproduced by an engineered manufacturing process from a set of similar category primary securities. For instance, for a bond, the replicant will be composed from other bonds of comparable crédit risk and feature sets, so as to provide critical identity to the target, such as cashflows which match the target bond's, both in amount and in the timing of cashflow receipt.

As an example, assume a Treasury note matures in exactly two years. A Treasury security in general is of the highest credit quality, which is equivalent in quality only to other Treasury securities. The holder of this Treasury will receive interest payments very six months until maturity, plus the redemption of principal to par on the final payment date. This cashflow sequenc is another key identity of Treasuries, along with the facts that Treasuries are devoid of optional features (early call or sinking fund provisions) and are priced according to unique determinants, such as proportionment of annual time increments at actual days divided by 365.

Let this two-year to maturity Treasury carry an interest coupon of 6.00 percent, payable on a semi-annual basis, and let the owner hold a face amount of this bond equal to $100 million. Thereby, in six months, the bond holder will receive $3 million, in one year, the holder will receiv another $3 million, in one-and-a-half years, the holder will receive a third interest payment of $3 million, and at two years, the holder receives $3 million in interest plus the redemption of principal at par of $100 million.

Given that this target instrument under discussion is a Treasury note, it is possible to replicate the amount and timing of cashflows, without alteration of the credit quality, through alternative Treasury instruments, assuming these latter Treasuries provide cashflows on the dates payable by the target Treasury. It is possible to replicate the target's cashflows with either other coupon-bearing Treasury notes or bonds, or with zero-coupon Treasury STRIPS. These methods are detailed below.

A. Alternate Treasury Notes/Bonds:

Assume the following availability of other Treasury Notes or Bonds:
1) Treasury Note, maturing in six months, carrying a 5.00% coupon;
2) Treasury Note, maturing in one year, carrying a 5.50% coupon;
3) Treasury Note, maturing in 1.5 years, carrying a 7.00% coupon;
4) Treasury Note, maturing in 2.0 years, carrying a 4.50% coupon.

To compose a replication with these four issues, one constructs four equations, and solves them simultaneously to achieve basket face amounts for each issue. Each equation is set equal to the cash flow to be received from the target bond, and each issue is weighted by the amount of cashflow receivable from it at that timeframe: Formula 2.1:

Equation	1)		2)		3)		4)	=	T)
a)	1.025 × 1)	+	0.0275 × 2)	+	0.035 × 3)	+	0.0225 × 4)	=	3.00
b)	0 × 1)	+	1.0275 × 2)	+	0.035 × 3)	+	0.0225 × 4)	=	3.00
c)	0 × 1)	+	0 × 2)	+	1.0375 × 3)	+	0.0225 × 4)	=	3.00
d)	0 × 1)	+	0 × 2)	+	0 × 3)	+	1.0225 × 4)	=	103.00

Solving for these equations gives the following face amounts per issue 1) - 4):
1) = $672,959 2) = $689,783 3) = $706,984 4) = $100,733,496.

Thus, in the case of this specific target Treasury, a replicated equivalent can be manufactured by purchasing the set of Treasuries 1) - 4), in face amounts of $672,959, $689,783, $706,984 and $100,733,496, respectively. When packaged together, these four Treasuries constitute a "replicated equivalent primary security" for the target Treasury.

This manufactured financial instrument can be marketed as an alternative Treasury offering into the marketplace, equivalent in substance (ie. it is a primary security, though manufactured by an engineered composition), credit quality and cashflow to the target. It is not a derivative security because its value is not derived from some other underlying instrument, and it is not de facto the same as the target primary security.

Another means to create a replicated equivalent for the example Treasury Note is to use Treasury STRIPS, which are zero-coupon instruments. While these types of Treasuries do not have coupons, they share the same credit quality as the target, and nontheless, can be manufactured in a way as to provide an investor the cashflows which are identical to the target Treasury coupon-bearing Note.

This method is quite simple, though the STRIPS do not pay out interim coupon cashflows. Rather, a STRIP is dedicated to each of the cashflows of the target for each of its cashflow periods: Formula 2.2:

1) six-month zero, face value $3 million
2) one-year zero, face value $3 million
3) 1.5 year zero, face value $3 million
4) two-year zero, face value $103 million.

The remainder of this section configures a couple of replicant possibilities for a target Treasury issue. The goal of this exercise is to demonstrate that, while the replicated equivalent can be manufactured to be identical to the target security's cashflows for instance, slight differences will arise in pricing and price sensitivities (duration and convexity, per S.3, S.4 or per 1.3, 1.4, or per 1.3a, 1.4a).

This is of significant value to the investment community because through replicated equivalent primary securities, the investor can choose from among an expanded set of alternatives, some of which will offer more optimal features, such as price improvements or more favorable pricing sensitivity, without any difference in security type or credit quality, and without resorting to derivative financial instruments.

Using the same time frame as the testing which accompanies the analytic invention section, Spring 1996, the pricing environment for Treasuries had been declining. Hence, it can be mentioned that equivalent securities which had reduced sensitivity to the increases to the yield curve, ie. are more resistant to price reduction, ie. had lower duration values, would be desirable to investors seeking to protect the value of their holdings.

As shown in the examples which follow, it is possible to manufacture replicated equivalent primary securities which provide this beneficial pricing sensitivity, and even, to manufacture replicants with favorable sensitivity improvements at a cost less than the target security.

Target Security

As based upon closing prices for April 3, 1996, the test creates replicants for the following Treasury Note, maturing 5/99, trading near par:

Maturity:	May 1999
Coupon:	6.75% s.a.
Yield:	5.94%
Prices: Bid/Ask	102:07;102:07 / 102:09;102:11
Face Value:	$50 million
Best Price:	$51,140,625
Accrued Interest:	$1,300,205
Total Cost (P+AI):	$52,440,830
Modified Duration:	2.782972

Replicant A: (using intermediate T-Notes)

Maturity	5/96	11/96	5/97
Matur, yrs fr 4/3	.114754	.614754	1.114754
Coupon	7.375%	7.25%	8.50%
Cheapest Ask Yield	4.46%	5.28%	5.48%
Bid Prices	100:07;same	101:03;same	103:04;103:04
Ask Prices	100:09;:10	101:05;:06	103:06;103:07
Basket Coefficient	-0.8895348	-0.9222442	-0.95576775
Face Value	($444,767)	($461,121)	($477,887)
Best Price	($445,740)	($466,165)	($492,821)
Accrued Interest	($12,636)	($12,968)	($15,649)
Total Cost	($458,376)	($479,133)	($508,470)
Duration	(0.113957)	(0.597015)	(1.059356)
Convexity	(0.068846)	(0.647529)	(1.651005)

Maturity	11/97	5/98	11/98
Matur, yrs fr 4/3	1.614754	2.114754	2.614754
Coupon	8.875%	9.00%	8.875%
Cheapest Ask Yield	5.71%	5.78%	5.85%
Bid Prices	104:23;same	106:07;same	107:03;02
Ask Prices	104:25;:27	106:09;:11	107:03;06
Basket Coefficient	-0.9963879	-1.0406026	-1.0874297
Face Value	($498,193)	($520,302)	($543,715)
Best Price	($521,702)	($552,658)	($582,285)
Accrued Interest	($17,034)	($18,040)	($18,590)
Total Cost	($538,736)	($570,698)	($600,875)
Duration	(1.500120)	(1.923568)	(2.334071)
Convexity	(3.035738)	(4.776208)	(6.855101)

Maturity	5/99
Matur yrs fr 4/3	3.114754
Coupon	9.125%
Cheapest Ask Yield	5.94%
Bid Prices	108:27;26
Ask Prices	108:29;30
Basket Coefficient	98.864316
Face Value	$49,460,543
Best Price	$53,834,709
Accrued Interest	$1,737,723
Total Cost	$55,572,432
Duration	2.718305
Convexity	9.188165

Values for Replicant A:

Total Cost:	$52,416,144
Duration:	2.610444

This replicated equivalent primary security allows the target Treasury's basket of cash flows to be purchased at a slightly lower total cost ($52,416,144 vs. $52,440,830), while also providing a significant reduction in duration (2.610444 vs. 2.782972). Replicant A demonstrates the positive value to investors which these products afford the investor.

Replicant B (Zero-STRIPS)

Maturity	5/96	11/96	5/97
Coupon	None	None	None
Yield	5.20%	5.31%	5.55%
Bid Prices	99:15;99:15	96:27;96:28	94:04;94:04
Ask Prices	99:15;99:15	96:28;96:29	94:04;94:05
Face Value	$1,687,500	$1,687,500	$1,687,500
Total Cost	$1,678,535	$1,634,766	$1,588,359

Maturity	11/97	5/98	11/98
Coupon	None	None	None
Yield	5.73%	5.82%	5.90%
Bid Prices	91:10;91:10	88:19;88:19	85:28;85:30
Ask Prices	91:11;91:11	88:20;88:20	85:30;86:00
Face Value	$1,687,500	$1,687,500	$1,687,500
Total Cost	$1,541,426	$1,495,547	$1,450,196

Maturity	5/99
Coupon	None
Yield	5.95%
Bid Prices	83:08;83:10
Ask Prices	83:10;83:12
Face Value	$51,687,500
Total Cost	$43,062,148

Values for Replicant B:

Total Cost:	$52,450,977
Duration:	2.828008

The STRIPS-based replicaant duplicates the target issue's cash flows for a higher total cost ($52,450,977 vs. $52,440,830), and is accompanied by a higher duration as well (2.828008 vs. 2.782972). This replicated equivalent does not provide the investor any improvement, given the contemporary environment, since it is more expensive than the target security and has pricing sensitivity which will make it tend to lose value more easily than the target as well.

Replicant C (using intermediate T-notes)

Maturity	5/96	11/96	5/97
Matur, yrs fr 4/3	.114754	.614754	1.114754
Coupon	7.375%	7.25%	8.50%
Cheapest Ask Yield	4.46%	5.28%	5.48%
Bid Prices	100:07;same	101:03;same	103:04;103:04
Ask Prices	100:09;:10	101:05;:06	103:06;103:07
Basket Coefficient	-0.91302988	-0.9466032	-0.98101223
Face Value	($456,515)	($473,301)	($490,506)
Best Price	($457,514)	($478,478)	($505,834)
Accrued Interest	($12,970)	($13,219)	($16,062)
Total Cost	($470,484)	($491,697)	($521,896)
Duration,mod	(.113953)	(.597015)	(1.059356)
Convexity	(.068843)	(.647529)	(1.651005)

Maturity	11/97	5/98	11/98
Matur, yrs fr 4/3	1.614754	2.114754	2.614754
Coupon	8.875%	9.00%	3.50%
Cheapest Ask Yield	5.71%	5.78%	3.08%
Bid Prices	104:23;same	106:07;same	100:01;99:18
Ask Prices	104:25;:27	106:09;:11	100:01;100:18
Basket Coefficient	-1.0227052	-1.06808779	-1.1161517
Face Value	($511,353)	($534,043)	($558,076)
Best Price	($535,482)	($567,254)	($558,250)
Accrued Interest	($17,483)	($18,516)	($7,525)
Total Cost	($552,965)	($585,770)	($565,775)
Duration, mod	(1.500120)	(1.923568)	(2.507384)
Convexity	(3.035754)	(4.776208)	(7.597885)

Maturity	5/99
Matur yrs fr 4/3	3.114754
Coupon	9.125%
Cheapest Ask Yield	5.94%
Bid Prices	108:27;26
Ask Prices	108:29;30
Basket Coefficient	98.864316
Face Value	$49,460,543
Best Price	$53,834,709
Accrued Interest	$1,737,723
Total Cost	$55,572,432
Duration, mod	2.716745
Convexity	9.182892

Values for Replicant C:

Total Cost:	$52,383,845
Duration:	2.603796

Replicant C reduces the total cost of purchasing identical cash flows substantially ($52,383,845 vs. $52,440,830, ie. 11 b.p.). Plus this replicant provides the lowest duration of any of the alternative Treasury purchases. This is a clear win/win optimization over the target security, because the purchase price of C is cheaper than target security, while also providing a pricing sensitivity which provides valuable price protection against the present environment of increasing yields.

Overall, the samples in this section positively affirm the usefullness of manufactured replicants within the investment community, because they can provide opportunities to purchase equivalent cashflows with more desirable sensitivities, often at a total cost less than the cost of the less desirable target security.

In summation, manufacturing replicated equivalent primary securities can provide the investor with a means to broaden the palette of comparable alternatives to any given primary security, without changing the credit quality or other features, and without employing derivative instruments. Thus, they augment the set of investment choices within the primary securities by offering variations on pricing and pricing sensitivity without introducing the uncertainties of derivative instruments.

This final section of the invention description discusses the arbitrage, trading and sales aspects relating to the prior two components of the invention: the analytic methods and process, and the engineered manufacture of replicated equivalent primary securities.

"Arbitrage" can be generally of two types:

> "pure" arbitrage, that is, the two sides to the transaction create a riskless neutrality, where the traded positions are equivalent positions using identical instruments; and
>
> "relational" arbitrage, where the positions stand in an identifiable necessary interrelation, such as a Treasury versus a Treasury future.

For the purposes herein, both types of arbitrage can be effected using the results of the invention to this point. In the case of the replicated equivalent primary securities, one can effect a pure arbitrage between a replicant and a target security, or between two identical replicants if and when the pricing between these two securities shows a spread differential which allows a profit to be captured by selling one of the securities and buying the other.

From the examples of replicants already supplied, one simply compares the purchase prices of the various alternatives (ie. the target, and replicants A, B, and C) against the selling price for these instruments, and determines the spread differential which can be captured. In the operational business context, the costs for transacting these positions, as well as any costs of carry for related financing are deducted from the gross spread to establish the net profitability.

For the example herein, the gross spreads are examined, since transaction and finance costs vary according to the participant seeking to enact the arbitrage transaction. The following grid shows the purchase and selling prices for the target and its three alternate replicants:

<u>Values of Reverse Positions of Target, Replicant A, B, and C:</u>

Target	can be sold for $52,409,580	and bought for $52,440,830
A	can be sold for $52,383,749	and bought for $52,416,144
B	can be sold for $52,450,920	and bought for $52,450,977
C	can be sold for $52,351,321	and bought for $52,383,845.

A number of arbitrage opportunities are indicated from the above prices of buying or selling the target and sample replicants. The most attractive entails buying C and selling B, which shows a gross arbitrage spread of 12.8 b.p. (0.128%). Also, one could buy A and sell B, securing 6.6 b.p., or one could buy C and sell Target, grossing 4.9 b.p. profit.

Relational arbitrage can also be effected using the results of the invention to this point. Specifically, the analytic processing showed how to establish the governing yield, and how to compare th yield to the market spot yield (in the example, the Treasury versus the spot Treasury STRIPs yield). The processing showed moreover that differentials exist between these two yields, and that a reversional tension exists to keep these yields in line with each other, restoring the spread to a marginal basis when it opens up a divergence. This forc underlies the lock and operates over short periods, ie. one month.

Using that knowledge, the arbitrageur can profit by buying whichever of these two securities has the lower price (the higher rate) and selling the security with the higher price (the lower rate). Again, these are gross spreads, with actual net results depending upon the circumstances of the individual trader. Nontheless, the means are established to perform this type of arbitrage trade. In the example, the portfolio of Treasuries has a higher rate, thus is bought, while selling the spot STRIP Treasury.

As mentioned in the Prior Art section, the forward rates are more akin to an option embedded in the yield structure then as a true rate curve. This assertion is also supported by the fact that the forward curve is discontinuous, and displays properties of the resetting feature found in futures as the spot contract expires and the next contract becomes the present spot contract.

Consequently, the arbitrage engine can make use of this discovery, and works to compare the spread which can be realized between offered forward rates (whether as direct quotes, or by extraction from the term structure) and relevant relational vehicles (such as rate futures and options).

The simplest method to capture the forward rate premium is to sell the forward rate and take a short position in the relevant futures contract (assuming the futures is priced at discount to par, rather than as directly in terms of rate), employing a stop-loss feature to the future in the event that this contract begins to rise. In this manner, the arbitrageur will net out the premium by expiration when the forward rate returns towards the spot rate for that term or will realize a net-zero position if the spot rate indeed rises to or above the rate of the forward at the initial time of instituting the abritrage positions.

In similar manner, the arbitage engine will compare the profit realizable by using an option contract corresponding to the forward's time frame. In this case, the option will not require a stop-loss trigger, since gains flow through to this vehicle, however, this type of abitrage lock is dependent upon the level of premium applicable to entering the option position, as well as requiring that the directionality of option payout is not engendering a prospect of unlimited downside exposure. Hence, a long put would be typically used.

For long-span forwards, the arbitrage engine can effect a stack-and-roll position in the futures or options. However, these strategies are inherently variable in outcome, and can be costly in terms of maintenance or premiums. Hence, were these to be used, then the engine would seek to unwind the arbitrage positions prior to terminal expiration, when the profit realizable reached a specified level, or at the command of the end-user.

Consolidated position tracking and monitoring is a necessary part of the system, both for the arbitrage trading, but also for the manufacuring and sales activity effected by the replicant generator. Because replicated equivalent primary securities have not been manufactured for sale into the financial markets, the invention entails that this manufacture for market is a component of this invention's specification. Indeed th manufacture, issuance, trade and arbitrage of these securities are properly an aspect of this invention's specification.

SYSTEMS ARCHITECTURE

In all cases, whether for th analytic processing, the manufacture and delivery of replicated equivalent primary securities, or for the assessment of arbitrage spreads, and the enaction of arbitrage transactions, replicant issuance and trading, the system appropriate for these ends are an integrated computer-based information system.

Given that arbitrage is a calculative endeavor, and that the pricing within financial markets is swift and changeable, the enactment of arbitrage transactions is uniquely suited to computer-generated operation. A data-feed can be linked into the computational engine, which searches for viable trades, and upon identifying such, effects the position via automated execution. In fact, all of the core engines are fed by real-time signals.

It is computational processing rather than intelligent processing, taking maximal advantage of accelerated computation and automated input-output, while avoiding dangers of artificial intelligence or out-guessing the market. Moreover, such a computerized system can reliably function in the absence of regular human supervision, and on an uninterrupted, 24-hour, global trading basis.

While the calculation of analytics and the engineering of replications are intensive on-going, real-time activity, these tasks can be handled by today's high-end workstation PC's, and readily handled by mini-computers or mainframes. Whether linked to commercial data-feeds or, as within the institutional setting of the inter-dealer Treasury screen, the computerized valuation of securities and assemblage of replicants, and the manufacture over market channels, trading, delivery and arbitrage based thereon are well suited to present computer technology and information systems.

The general system solution is to compose the business logic (the computational engines) as three core systems: the analytic valuation processing engine; the replicant generator; and the automated arbitrage engine. These three components are situated on individual server computers or as three independent virtual servers on one computer platform. They are interlinked as necessary, and each of these core components are backed up by storage medium, and are addressable by input devices, with output routed to user display, to external destinations and to further processing.

Each of the components receives its data through a real-time financial data feed, and the relevant signal data (ie. for analytic valuation: security type, credit rating, coupon, maturity, price) is delivered as arrays for computational processing. The output of the analytic engine and replicant generator are sent into the arbitrage engine for processing there, as well as to displays or external destinations necessary for functioning, manufacture, monitoring and control.

Output from any of the engines can be sent to terminals and printers, and each of the engines must be linked to storage medium allowing output and results to be stored to memory, so as to allow monitoring, review and assessment of the operating systems, trades, sales, inventory and P&L.

Automated control sequences are to be established, particularly to accomplish computer-driven transactions and automated assembly of replicants which are effected over communications lines with securities dealers, brokers and investors. Given the depth and magnitude of engineered processes, the computations performed within the engines must be integrated within a computerized information system since real-time operation is the underlying environmental constraint.

Given the sophisticated financial services which are being provided herein, it is necessary to secure the integrated system against unauthorized intruders and vandals, so encryption designs and gate-keeper devices are warranted. Moreover, subject to any legal requirements of the operational system, firewalls may be demanded between core processes, such as the segregation of replicant generation for external investor-clients from the automated arbitrage engine for internal positions by the end-user.

PORTFOLIO OF SECURITIES using bloomberg closing bid prices, quotes and dates
Treasuries

3/22/96

Form. 1.2b Form. 1.3 Form. 1.4

Security	FV $MM	Bid Price	Maturity	Coupon	YTMprint	YTMprbonDZ	YTM	perann	AI	Full Value	PortCoef	YrstoMat	DurMod	BonKDZY	Convexity	BonVdzy	PortCnoAI
T-Note	70	99.34375	11/15/96	0.043875	0.0542	0.054254	0.054254	2	1.077041	70.61767	0.137015	0.652055	0.632784	-0.62863	0.708984	0.704331	0.138776
T-Note	100	100.5625	5/31/97	0.06125	0.0562	0.056304	0.056304	2	1.896233	102.4587	0.198795	1.191781	1.135673	-1.14206	1.855837	1.866276	0.197791
T-Note	40	100.0313	10/31/97	0.0575	0.0572	0.057294	0.057294	2	0.901096	40.9136	0.079382	1.610959	1.518323	-1.5188	3.083305	3.084268	0.078698
T-Note	120	100.0625	8/15/98	0.05875	0.0584	0.058467	0.058467	2	0.695342	120.7703	0.234324	2.4	2.20918	-2.21056	6.110799	6.114618	0.236169
T-Note	40	99.90625	3/31/99	0.05875	0.0591	0.059093	0.059093	2	1.120274	41.08277	0.079711	3.024658	2.733996	-2.73143	9.131777	9.123216	0.0786
T-Note	80	99.25	6/30/00	0.05875	0.0607	0.060768	0.060768	2	1.068767	80.46877	0.156129	4.276712	3.729308	-3.70481	16.65721	16.53228	0.156168
T-Note	60	98.125	2/28/01	0.05625	0.0607	0.060698	0.060698	2	0.212671	59.08767	0.114644	4.942466	4.247682	-4.16804	21.50725	21.10399	0.115798
																	1
Total	510	508.4281						2	6.971424	515.3995	1						

4/3/96

Security	FV $MM	Bid Price	Maturity	Coupon	YTMprint	YTMprbonDZ	YTM	perann	AI	Full Value	PortCoef	YrstoMat	DurMod	BonKDZY	Convexity	BonVdzy	PortCnoAI
T-Note	70	99.375	11/15/96	0.043875	0.0542	0.054276	0.054276	2	1.174795	70.7373	0.136986	0.619178	0.601314	-0.59756	0.654654	0.650583	0.136804
T-Note	100	100.5625	5/31/97	0.06125	0.0561	0.056169	0.056169	2	2.091872	102.6544	0.198795	1.158904	1.10543	-1.11165	1.771985	1.781952	0.19777
T-Note	40	100.0313	10/31/97	0.0575	0.0573	0.05729	0.05729	2	0.974044	40.98654	0.079372	1.578082	1.488391	-1.48888	2.976234	2.977164	0.07869
T-Note	120	100	8/15/98	0.05875	0.0586	0.05875	0.05875	2	0.92459	120.9246	0.234176	2.387123	2.181091	-2.18109	5.966026	5.966026	0.235997
T-Note	40	99.84375	3/31/99	0.05875	0.0593	0.059328	0.059328	2	1.194262	41.13176	0.079654	2.991781	2.70671	-2.70248	8.95829	8.944293	0.078543
T-Note	80	99.34375	6/30/00	0.05875	0.0605	0.060527	0.060527	2	1.219945	80.69495	0.156269	4.243836	3.704021	-3.67971	16.43766	16.32979	0.156299
T-Note	60	98.21875	2/28/01	0.05625	0.0604	0.060498	0.060498	2	0.322746	59.254	0.114748	4.909589	4.225993	-4.15072	21.27874	20.89971	0.115897
																	1
Total	510	508.4813		CF received,invested:			+	2	7.902254	516.3835	1						

4/25/96

Security	FV $MM	Bid Price	Maturity	Coupon	YTMprint	YTMprbonDZ	YTM	perann	AI	Full Value	PortCoef	YrstoMat	DurMod	BonKp	Convexity	BonVp	PortCnoAI
T-Note	70	99.40625	11/15/96	0.043875	0.0545	0.054807	0.054807	2	1.359406	70.94378	0.137482	0.558904	0.543475	-0.54025	0.559982	0.556857	0.137416
T-Note	100	100.4063	5/31/97	0.06125	0.0572	0.057381	0.057381	2	2.460041	102.8663	0.199345	1.09863	1.049362	-1.05362	1.621102	1.627688	0.198283
T-Note	40	99.84375	10/31/97	0.0575	0.0585	0.058591	0.058591	2	1.112295	41.0498	0.079551	1.517808	1.433287	-1.43105	2.782871	2.778523	0.078869
T-Note	120	99.625	8/15/98	0.05875	0.0604	0.060516	0.060516	2	1.348361	120.8984	0.234289	2.306849	2.126901	-2.11892	5.693688	5.672337	0.236088
T-Note	40	99.34375	3/31/99	0.05875	0.0612	0.06123	0.06123	2	1.335519	41.07302	0.079596	2.931507	2.653679	-2.63626	8.627816	8.571196	0.078474
T-Note	80	98.5625	6/30/00	0.05875	0.0627	0.06271	0.06271	2	1.502459	80.35246	0.155715	4.183562	3.651503	-3.59901	15.98882	15.75898	0.155714
T-Note	60	97.1875	2/28/01	0.05625	0.0631	0.063072	0.063072	2	0.525615	58.83812	0.114023	4.849315	4.169654	-4.05238	20.74093	20.15759	0.115156
																	1
Total	510	506.3781		CF received, invested:			+	2	9.643696	516.0218	1						

Aggregate Values for Portfolio

DATE	3/22/96	4/3/96	4/25/96
Price	0.996918	0.997022	0.992898
Coup	0.056823	0.056823	0.056819
YTM	0.057971	0.057956	0.059529
DZY	0.058032	0.057956	0.059529
Mat	2.491961	2.459635	2.394768
DurM	2.237812	2.21037	2.151482
BonK	-2.22571	-2.19835	-2.12691
Con	7.938994	7.799226	7.496867
BonV	7.872984	7.739256	7.385564

K and V(+DurM and Conv) calc from C, Y, T

	3/22/96	4/3/96	4/25/96
K DZY	-2.28548	-2.25715	-2.19219
K YTM	-2.28588	-2.25715	-2.19219
DurMod	2.1083	2.085019	2.03596
V DZY	6.521511	6.376812	6.056289
V YTM	6.522848	6.376812	6.056289
Convexity	6.128838	5.995918	5.724996

09489739.012100

Partial Differential Process and Algorithms for
Change (Δ or d) in Price (P) with respect to Yield and Time,
in Discrete Form: Formula 1.6:

$\Delta P = A + B + C + D$

where,

$\Delta P =$ change in bid price, for given changes in yield and time
$A =$ -(Duration) $\times$ Price(dirty) $\times \Delta Y$
$B = \frac{1}{2} \times$ Convexity $\times$ Price(dirty) $\times (\Delta Y)^2$
$C =$ Theta $\times$ Price(dirty) $\times \Delta t$, with $\theta = 2 \ln(1+r/2)$, $r =$ ytm
$D =$ -(Δ Accrued Interest, for given Δt),

with,

Y (ytm), computed on applicable day-count basis (Formula S.2)
Duration and Convexity, standard modified annualized (Formulas S.3 and S.4)
Theta (θ) recalculated at cash flow dates
Price (dirty) equals bid price plus accumulated interest
ΔP rounded to nearest 1/32[nd] per market price convention
Arbitrage differential of precise ΔP minus actual market price change.

The above process can be used for a single security or group thereof, each single security processed separately, or as a set of securities in aggregate weighted summation as below:

$\Delta Pp = Ap + Bp + Cp + Dp$

where p is on a portfolio basis, each security having a portfolio coefficient based on its portion of the present value, with invention's Aggregate Value Calculations for Portfolio utilized, establishing the Aggregate Values for Portfolio.

Example Spreadsheet (using Bloomberg prices and yield (ytm)):

Over Two Discrete Dates, Seven Treasury Notes, each held as $100 face value

		3/22/96							A+	B+	C+	D=	dP			
	Bid Price	Maturity	Coupon	AcInterest	Yield	Duration	Convexity	Theta	dDuration	dConvex	dTheta	dAcInt	dBid Price	Round dP	ActMrkt dP	ArbitDiffer
1	99.34375	11/15/96	0.043875	1.543	0.05421	0.6376	0.704	0.053488	-0.00257	5.68E-08	0.177411	-0.145	0.029838	0.03125	0.03125	-0.00141
2	100.5625	5/31/97	0.06125	1.901	0.05621	1.147	1.855	0.055435	0.015278	1.57E-08	0.18674	-0.202	1.88E-05	0	0	1.88E-05
3	100.0313	10/31/97	0.0575	2.259	0.05725	1.525	3.083	0.056446	0	0	0.189826	-0.189	0.000826	0	0	0.000826
4	100.0625	8/15/98	0.05875	0.581	0.05844	2.262	6.111	0.057602	-0.06147	2.23E-07	0.190597	-0.1937	-0.06457	-0.0625	-0.0625	-0.00207
5	99.90625	3/31/99	0.05875	2.808	0.05909	2.739	9.131	0.058234	-0.06752	2.63E-07	0.196651	-0.17792	-0.04879	-0.0625	-0.0625	0.013711
6	99.25	6/30/00	0.05875	1.323	0.06074	3.792	16.657	0.059836	0.091529	4.8E-07	0.197848	-0.194	0.095378	0.09375	0.09375	0.001628
7	98.125	2/28/01	0.05625	0.34	0.06069	4.361	21.507	0.059787	0.090175	4.74E-07	0.193544	-0.1854	0.09832	0.09375	0.09375	0.00457
		4/3/96														
1	99.375	11/15/96	0.043875	1.688	0.05425											
2	100.5625	5/31/97	0.06125	2.103	0.05608											
3	100.0313	10/31/97	0.0575	2.448	0.05725											
4	100	8/15/98	0.05875	0.7747	0.05871											
5	99.84375	3/31/99	0.05875	0.04842	0.05933											
6	99.34375	6/30/00	0.05875	1.517	0.0605											
7	98.21875	2/28/01	0.05625	0.5254	0.06048											

In each of the seven fixed-income securities, the process returned values in accord with the market change in price. Because up to 1/64[th] of one percent is under/over-valued relative to the 1/32[nd] notching, a premium can be gained. The process is apt to valuations and risk, as data systems typically compute standard yield, duration and convexity. The process provides for the estimation of analytic values, useful in fund, portfolio, trades and hedge management.

Portfolio Method for Fixed-Income Mutual and Hedge Funds

Cause

In addition to the cause for proprietary in-house investment management and analytic technologies, these adding profitability and security to in-house trading, investment and insurance portfolios, the market is in need of superior fixed-income funds. Institutional investors seek expert financial portfolio methodologies.

Departure

Assuming the short-term U.S. Treasury Bills as credit risk-free returns for fixed-income investment, the conservative long U.S. Treasury fixed-income fund manager enhances expected returns by taking on risk as lengthier expirations, yield curve weightings, speculative tactics and credit spread on U.S. agency paper.

My independent study of the mutual fund industry, for the sector dedicated to U.S. Treasury funds, showed this sector earning returns in excess of shortest-term U.S. Treasury paper. Yet unlike tactics above, the study showed that the aggregate sector performance approximated, and could be mirrored by, randomly selected, roughly evenly distributed, ladder index portfolios of Treasuries over short to mid term expirations.

The reward to risk by the Treasury mutual and hedge fund sector can be replicated solely by taking on moderated maturity risk. This is a good departure, since it can be earned without chance, being related to Malkiel's finding that returns by managed equity are similar in performance to that by random selection.

Fundament

The centerpiece of the methodology is a ladder-based U.S. Treasury portfolio, comprised of coupon-bearing Notes and Bonds spanning the short and medium terms. This composition can be expected to earn approximately the return of the mutual fund industry sector, but without managerial or credit risks.

Moreover, by virtue of balanced capital over a maturity spectrum, these portfolios being generally out to five, seven and ten year's maturity, the structure has a moderated yield-curve risk, a higher return to variance, a natural awareness of risk-points and a reduced reinvestment risk. They stay out of the shortest maturities, since, soon after start-up, they de facto maintain the shortest maturities paying yesterday's Note. When one instrument matures, a new one in the ladder, at the longest term, is bought with par face value.

The skilled management of these portfolios may appear easy, and that is because the character of the portfolios can be so readily approached. The intricacies of optimization, of loss avoidance, profit capture and of continuous perfection of the investment composition, are also present herein with equal opportunity.

Enhancement

The first enhancement has already been discussed - the arbitrage to mutual fund sector performance accomplished by reduction of assumed risk. The second enhancement is the group of sensitive managements to the exact portfolio holdings over time. The final two enhancements are similarly derivative-free arbitrages.

Remembering that any coupon-bearing Note or Bond is itself a fixed ladder portfolio of cashflows, and also that such cashflows are zero STRIP equivalents, the invention's methods focus on composition, replications and permutations which, by automated computerization, price out arbitrages on cashflows, represented or divisible within holdings and as against replications and constructions to be traded into hand.

Upon such discovery, this arbitrage does not change the certain future cashflows, but rewards the swap with serial and liquidity based profits. The final arbitrage is risk-free basis, capturing the reversional pricing dynamic between a ladder portfolio and its aggregate made by invention's governing yield processes.

Section II

Introduction

The insured deposit losses and insured catastrophe losses have been consequential and costly. Unlike most business scenarios, the end-guarantor on the higher portion of these losses has been the general tax-payer. For the commercial depository banks, the public sponsor is the Federal Deposit Insurance Corporation, an agency legislated into creation in 1933. Explicitly, this U.S. Federal institution stands as guarantor. Importantly, note that the FDIC insures the depositors, not the depository institutions.

The insured banks, and all financial holding institutions which have an insured bank within their structure, are burdened by substantial reporting requirements, outside supervisory monitoring and the threat of sanctions in the form of increased withholding requirements and restricted operations. Moreover, in the guise of FDIC "assessment rates", the insured banks pay fees to the FDIC, to cover banks failures and to build a capital surplus against the prospect of future closings, paying to insure the public accountholders.

Quizzical that the banks pay hefty "assessment income" to the FDIC, to fund bail-out of bank closure, without receiving direct protection in return. Disturbing too, that the present FDIC Bank Insurance Fund (BIF) stands at about $27 billion, only half of the amounts needed under historical crises. Recent mean annual deposit losses are less than half the BIF balance, with current annual figures under $1 billion.

The Federal Reserve Bank, the FDIC and the commercial banking industry are discussing the FDIC-insured banking structure and its future. The U.S. is alone among developed countries with this type of deposit guarantee, having the most segregated and legislated industry system as well. Conceivably, the deposit insurance program could end, enabling the long tentacles of bank regulatory requirements to be unwound from the financial institutions that are increasingly commingled in diverse financial industries.

Against losses from catastrophes, the final guarantor to property insurance is again the public, in the form of catastrophe relief from state and federal programs. Here too, the property/casualty insurance industry and its allied reinsurers gain no commercial relief or grant from the public mandate, for it is not the firms that are bailed out. Though the insurers do not pay a fee like the bankers to a regulatory agent, every catastrophe loss, even flooding which the Federal Government pays, entails related losses to insurers.

Now, both industries are regulated to observe public service laws that are sometimes uneconomic. The insurers are expected, even mandated under law, to provide property insurance that covers damages sustained in catastrophes, to a public which increasingly lives in catastrophe-prone zones. From 1988 through 1995, insured coastal property exposures increased over 80%, to a value of $3.4 trillion. By the year 2000, 75% of all Americans will live within ten miles of the ocean, $650 billion on New York coast.

Hurricane Andrew neutralized all annual income for the insurers and bit into their underwriting surplus. A single major catastrophe would thin the ranks of insurers, reduce the base for and interest in, insurance underwriting and could annihilate the entire catastrophe reinsurance industry, forcing a public bail-out of homes and businesses. Solutions have focused on municipal and Federal bonds, and cat options.

It is in the sensible interests of public and industry to consider feasible new means of mitigating these dollar risks. The cat derivative market has yet to gain substantial underwriters, though the demand for puts against loss is strong among insurers. The bond issues do not generally salvage the insurers that get devastated, nor does it capitalize thin reinsurance pools, and always these are adding to the public expense.

Specifically, the paper finds that the BIF balance is well above levels typically required for annual losses of insured deposits, but is well below the levels required in crisis situations. While the most recent years have had remarkably lowered bank closings, the underlying macro-economic analytics on the depository banking industry do not support the conclusion that the danger of crisis has been left in the past.

From the public perspective, acceleration in the build-up of the BIF balance is desirable. However, on a mean loss basis, the BIF balance can be considered to have substantial excess capital. Since the FDIC is a public entity, this surplus capacity can be mandated to address other public needs, such as excess of loss catastrophe reinsurance. By 'lending' out this surplus capital to insurers, the depository banks can gain advantage, with the insurers' payments to the FDIC reducing depository bankers' payment.

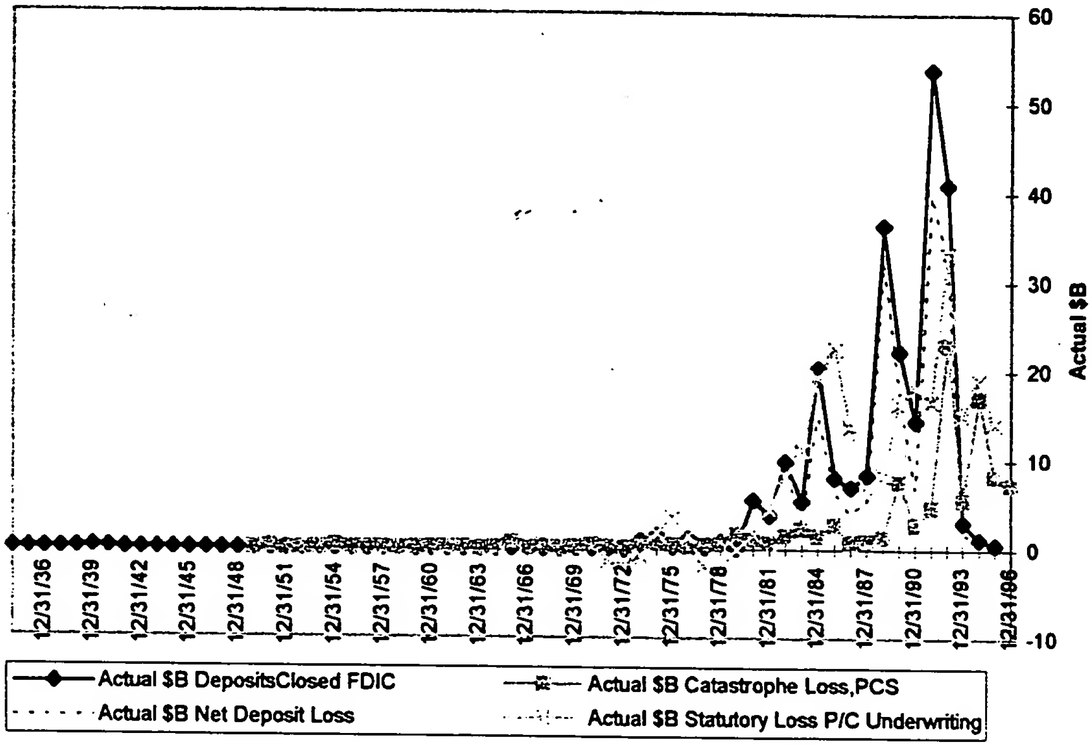
The evidence on catastrophe losses and potentials underscores a possibility that a mega-event could incapacitate the industry. On the other hand, recent events appear to be above the typical losses that one might expect on an averaged basis. Should substantial losses occur, in the absence of a commercially viable method of increasing the survival of the P&C insurers and reinsurers, then, not only will the public get the bill for the losses, but the P&C insurance industry's underwriting capacity would be much reduced.

We must consider whether there may be mutual and public benefits between the depository banking and insurance industries by enabling these industries to swap, option, pool or transfer portions of potential losses, utilizing the FDIC and/or its BIF as a central pivot. The insurance companies can evaluate the risks of depository bank default, or could reinsure the FDIC's portfolio, or parts thereof, or separated from the FDIC, through their methods of risk and data assessment, of reinsurance, contracts and of pricing.

This paper attempts to analyze the nature of deposit closings and of insured catastrophe losses as variables. It offers a methodology based upon these losses as being of theta, single state, variables, with eye upon single-variable derivatives, a swap or non-notional netting transactions. Additionally, it considers Merton's work on deposit guarantees and prefaces the context regarding reinsurance of the banks or FDIC.

The paper concludes with presentation of statistical values pertaining to the variables under study. The descriptive statistics, side-by-side, complement the macro-economic examination of historical loss experience and industry operation, serving as an initial departure towards the forecasting of expectations.

Actual $Billi n L sses,
U.S. Insured Deposits and Property & Casualty



The above chart shows the actual dollar figures for insured deposit closings, as documented by the FDIC, from its inception in 1934 to end-1995. The dollar amounts for the deposit losses, being the closed deposits minus recoveries by the FDIC that same year, are depicted for recent years. The above chart also sketches the insured catastrophe losses, as documented by Property Claims Services, the market reporter, from 1949 to present. Shown along with this is the net statutory loss of U.S. property & casualty insurers maintained by A.M. Best. All losses are as positive dollar values, with any gains being negative values.

Note two salient periods. There is clearly a major separation between the overall history, 1934 to present, dividing in the early 1970's. Prior to then, dollar losses are small and slow in rising. Afterwards, the pace of losses accelerates, with sharp rises in the amounts lost per annum. This division within the temporal fabric of the financial industries is expected, since the early 1970's ushered in 'modern' finance.

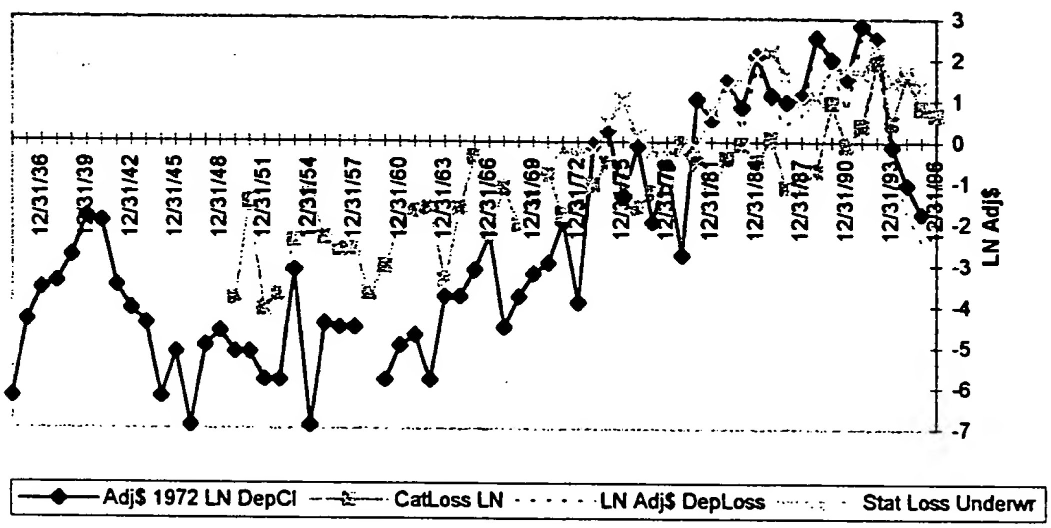
The watershed instigating 'modern' finance occurred circa 1972-4, coincident to the repeal of the Bretton Woods fixed exchange rates, that being initiated by the Nixon termination of the gold standard U.S. dollar. Non-physical contracts debuted, completing the revolution, as an obscure equity index listed on a U.S. mid-West exchange. By the 1980's, physicists model financial derivatives by heat transform formulae.

Yet behind the great transformations in the financial markets, industries and sciences lie a history which continues to impart profound complications upon our todays and futures but originates much earlier. Though our investment products and financial services interpenetrate simple divisions of industry groupings, the diverse operatives were mired in isolated struggle, hindered from sensible diversificati n and cooperation by the legislated separations of the Depression Era, unique market conventions or practices, industry rivalries or ignorance. Regulation, re-regulation and de-regulation are topics receiving attention.

One major consequence in the modern financial world, i.e. post fixed-exchange gold-standard, is volatility, which transforms stored value and discounted payoffs. At the core, this was monetary inflation, which arose like the tsunami by the end of the first 'modern' decade, jettisoning the stable rates of growth and losses, and with it, immediate actual dollar continuity with the previous forty years. Thus, in this research, all values past 1972 are adjusted by a 1972-dollar conversion, representing the inflation scalar, CPI. All values before 1972 are held as in original actual dollar amounts, on the fact that inflation impacted the financial markets on a very delimited basis between 1934 and 1971.

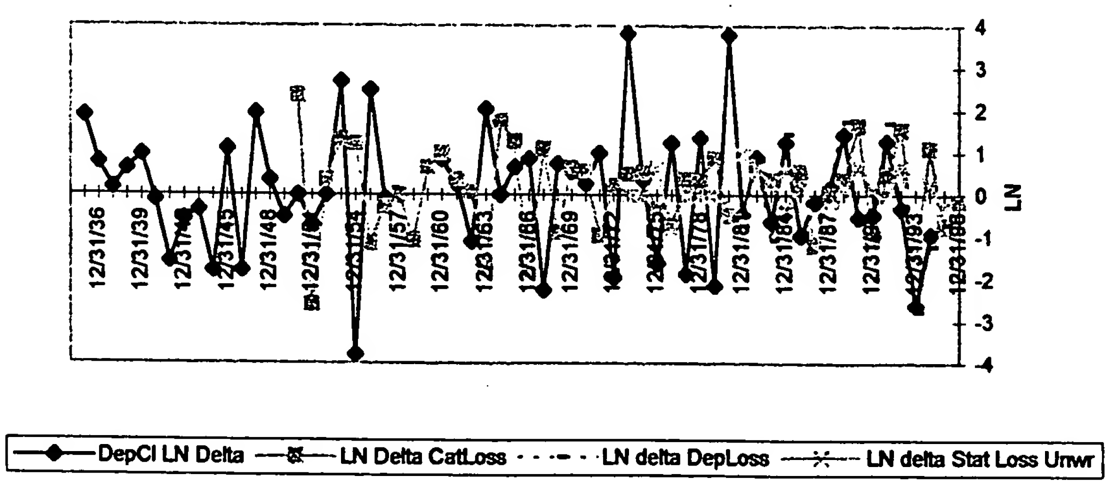
Before the specifics of either loss category are examined further, it is useful to perform logarithmic transformations on the adjusted-dollar data for the overall history. The chart below shows these values.

LN Adj-1972 Dollar Values, 1934-1996



The above chart shows the linear growth of the catastrophe losses and a undulating transform for the deposit losses. The two industries crossover in the 1970's, again in the most recent years. The chart below shows the volatility in movements from year to year, taking the natural log of the annual change for each adjusted dollar value. The deposit variables have the higher volatility of variance and range.

LN delta, Change in Adj$ Values, Year to Year



Next, each industry is examined separately. Each industry will be viewed for its own dynamic and characteristics. Operating factors effecting each industry will be reviewed, particularly for the 'modern' era, with findings, comments or conclusions being made. Later, methods and statistical data are presented.

This research is not investigating deposit losses or losses due to catastrophes in general. As long as banks and insurance houses have existed, there have been deposit and catastrophe losses. But U.S. deposit insurance was launched in 1934, precluding any prior existence of that specific variable. Catastrophe insurance in the U.S. was written as regular commercial insurance activity only after the second world war.

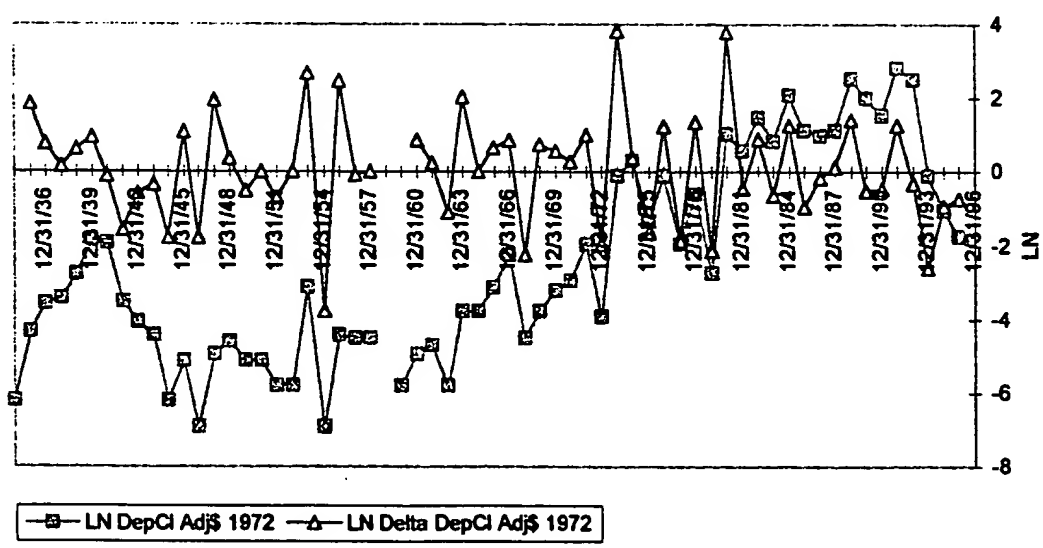
Thus, the research focuses on the U.S. insured losses of these types and their related industries over these entire histories. The consolidated U.S. property and casualty ("P&C") insurers industry data was available from PCS and A.M. Best, providing annual specifications for this research. As regards the values for the insured deposit variable, the FDIC is both industry insurer and consolidated data keeper throughout.

Part A: Insured Commercial, Depository Banking

The chart beneath displays the natural log values of the U.S. insured deposit closings, adjusted to 1972 dollar, for the entire life of U.S. insured depository banking. A simple linear band would capture most of the data throughout, but note that the first half dozen years, 1934-1940, contain the resolution of Depression Era bail-out. Thereafter, year-long low losses to the mid-Sixties. Then, sharp rises, cascading troubles befell the insured depository industry, commencing in the 1970's, growing until the early 1990's.

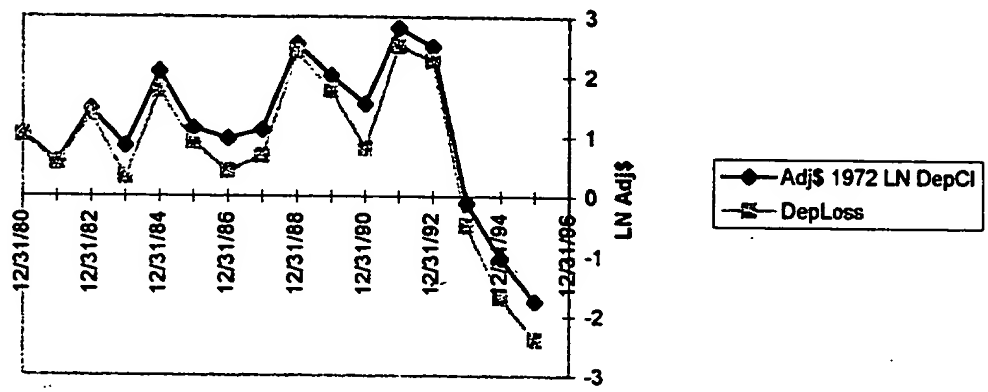
Most recently, a marked ebb, a seeming resolution to the torrent of the Eighties. It is crucial to note here that in1990, the FDIC segregated off the savings and loans into their own insurance fund, the Savings Association Insurance Fund. This fund is separate from the Federal Bank Insurance Fund, though also under the province of the FDIC. The BIF figures, being the data presented herein, are stripped thereafter of losses from this credit class. The SAIF has liabilities of $0.2 B, with fund assets of $3.8 B (year-end 1995 figures).

LN Values, Adj$ 1972, for Deposit Closings, 1934-1995



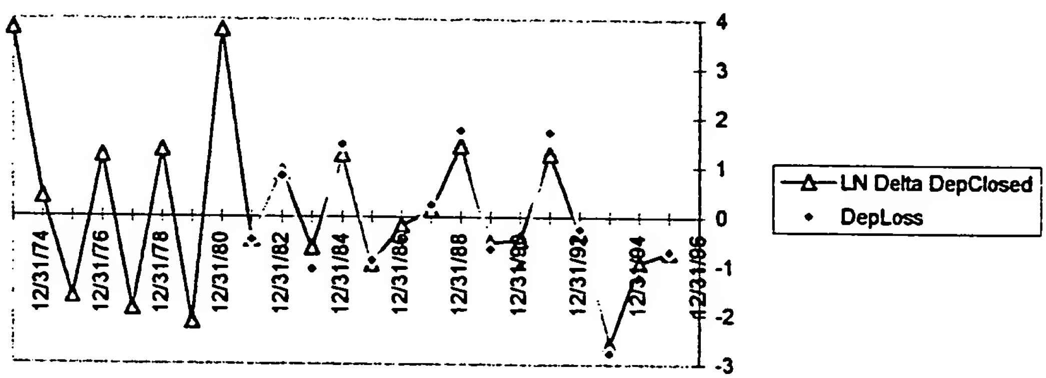
To provide perspective, examine the beneath chart, which maps the insured deposit losses against these insured deposit closures. The recovery on assets after closure only modestly reduces the magnitude of deposit closings, with some acceleration for persistence, when transforming 'closes' to 'losses' over time.

LN Dep sit Closings versus Dep sit Losse , 1980-1995



Examining the change in the log values for either deposit closings or deposit losses confirms that the effect of recoveries on the insured deposits is reasonably trivial to the undertaking. See the next chart.

LN delta Adj$ Values for Deposit Closings and Losses, 1972-1995

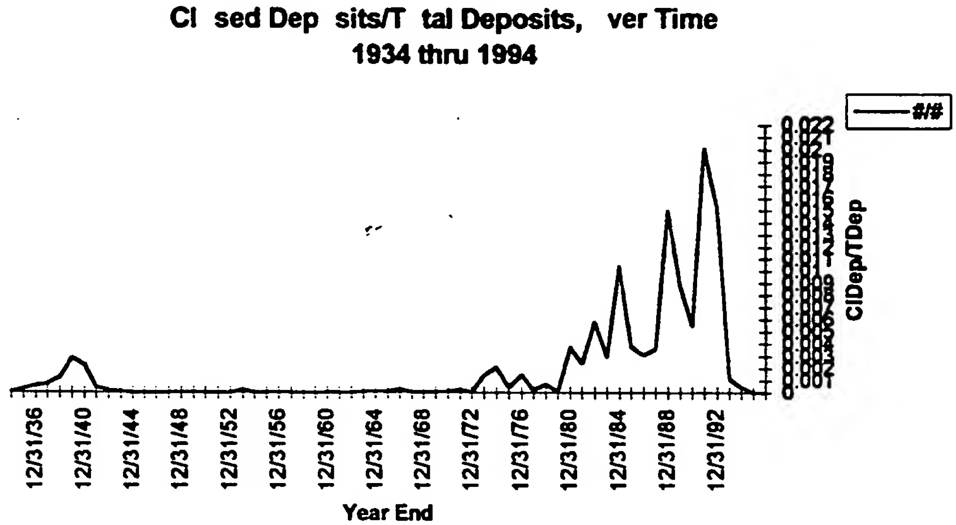


At the outset of operations, the FDIC charged the insured banks with an equal assessment rate of 0.0833%. This was continued unchanged throughout 1949, building up an overly substantial BIF surplus balance. The amount per account insured remained at its initial rate of $5,000 throughout this period as well. Thus, the FDIC covered ever less, since there was no upward adjustment for the economic growth, 1934-1949. In 1950, the amount per account insured was raised to $10,000, to $15,000 in 1965, $20,000 in 1969, to $40,000 in 1974, and finally to $100,000 in 1980, from hence it has not changed in actual dollars.

Yet for this increase in exposure covered after 1950, the effective assessment rate was radically lowered, insuring that as the fund continued, there was little new accumulation to pace the post-war economic growth. In 1950, the rate was cut to 0.0370%, lowered to 0.0323% in 1961, fluctuated between 0.0313% and 0.0333% until 1970, then stayed about 0.035% over the period to 1979 (0.0333%), 1980 (0.0370%). Finally, the rate was raised to 0.07% to 0.083%, 1981 to 1989, when it was abruptly raised to 0.12% in 1990, to 0.21-0.24% for 1991 through 1994. The rates were reduced for 1995 to 0.1240%.

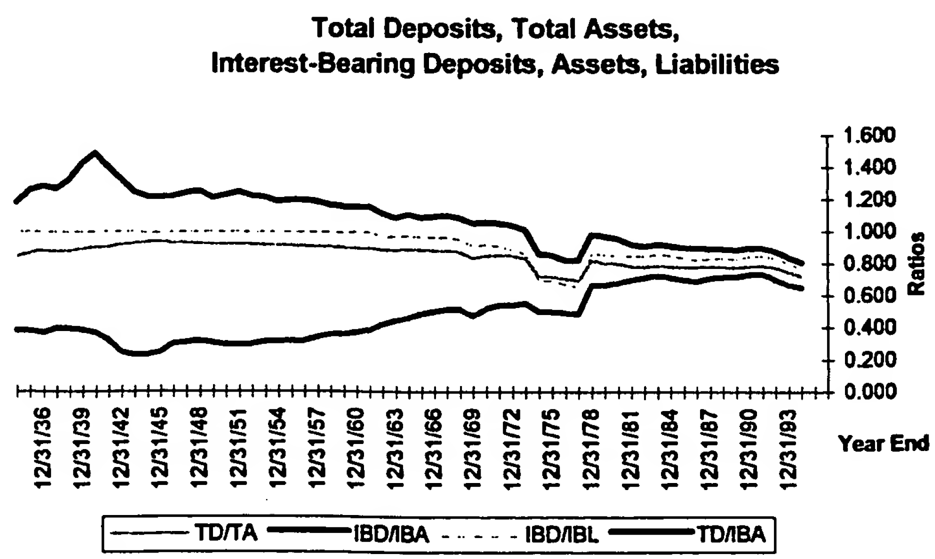
The dynamic is simple: the FDIC charges the banks a fee to build a surplus. This surplus is used to pay insured depositors at closed banks. The FDIC invests the surplus to provide for the future, using policy of assessment rates and insured account size to 'finesse' the assets and liabilities of the BIF. Under high closings, the banks pay increasingly greater fees to fund this operating expense, net of balance+recovery. When closings excede all of surplus+recovery+assessment fees, as in 1991, the public pays the remainder.

Before turning to the operating condition of the insured commercial depository banks, the focus is to establish how the insured deposit failures relate as a portion of total insured deposits over time. The chart below shows the historic ratio between actual dollar amounts of deposits closed vs. total insured deposits. This indicates, prime facie, that the "credit-quality" of the insured deposit banks has decreased markedly.



Failure during the 1970's followed on disintermediation of the savings function of depository banks into the securities houses, mutual and money-market funds. With the rise of interest rates, the banks lost customers since the assets to the banks of interest-bearing loans carried a term interest rate. In the 1980's, many real-estate and economic sectors collapsed, with wide slices of total asset value disappearing. These losses on mortgage and loan books, amidst towering assessment fees, increased the fail-rate, pushing deposit closing to historic high levels year-end1991. Since then, the closings have drastically abated.

Is it exhaustion before the next tirade, or is this the ushering in of a new financial era? A look at the depository banking industry's operating dynamic showcases the deepest truth about the 'modern' financial world: competition. The next chart contains bank operating ratios: operations are centered on narrow differentials. These figures reflect the condition of the total insured population, not just those of the closed banks. One senses how tense and demanding the operation of depository institutions has become..



Insured depository banks work between TD/IBA (total deposits over interest-bearing assets) and IBD/IBA (interest-bearing deposits/interest-bearing assets). In between these book-end signals run the rati s of IBD/IBL (interest-bearing deposits/interest-bearing liabilities) and TD/TA (total deposits/total assets), always in a definite order of, from top down, TD/IBA, IBD/IBL, (see 1974-77), TD/TA, then IBD/IBA.

It used to be that interest-bearing deposits were outweighed five-to-one by interest-bearing assets, IBD/IBA. This made the capture of an interest-rate spread virtually guaranteed. Under bank regulatory law, banks were prohibited from paying interest on demand accounts, only offering interest on term savings. In the latter 1960's to mid-1970's, restrictions were repealed, resulting in a recent ratio of only four-to-three.

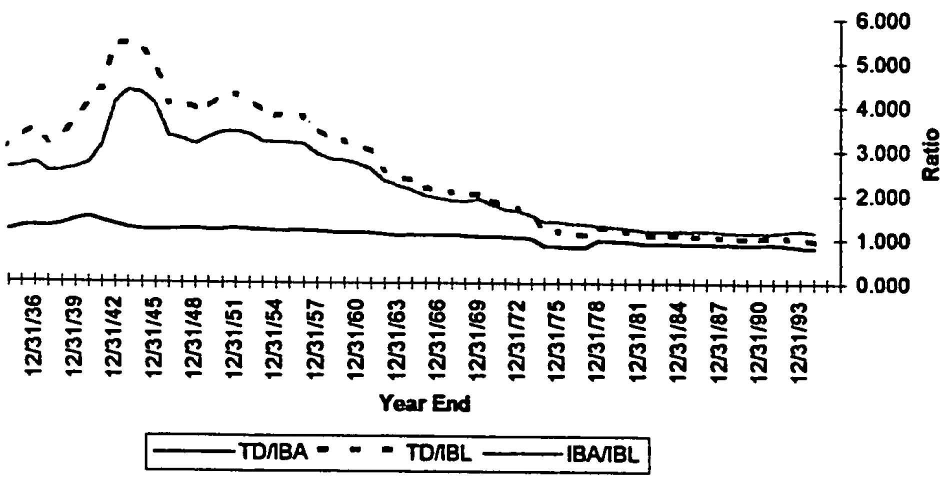
The upper bound of the operating channel is the TD/IBA ratio, total deposits to interest-bearing assets. This ratio shows values uniformly above one - up until the advent of the 'modern' era, circa 1972. Historic levels commence at 1.18 in 1934, rise steadily to 1.49 by 1940, then decline to a 1.20's range through 1943 to 1957. The descent thereafter is slow, finally breaching the threshold of unity, 1.01 in 1973, then plummeting under to 0.86 in 1974. It continues to sink, with a year-end 1994 ratio of 0.806. A TD/IBA ratio above one ensures that deposit solvency does not overly rely on rate-sensitive interest-bearing assets.

Betwixt these parameters runs the ratio, IBD/IBL, interest-bearing deposits to interest-bearing liabilities. During the years 1934 to 1961, this ratio ran precisely at unity, to within three decimal places (0.99x), each and every year. But then it fell, to 0.965 in 1962 and descended slowly if at all to 1969, when it dropped to 0.906, steady until 1972, when it then dropped under 0.9 to 0.85. By 1974, it fell to just under 0.7, continuing that pace, then rising into 0.8 to 0.85 before falling back down to 0.761, 1994. An IBD/IBL at one indicates that interest-bearing deposits are the only interest-bearing liabilities on the balance sheet.

Just under IBD/IBL ratio runs the total deposit to total asset ratio, TD/TA. This ratio is another indicator of the portion of deposit banking within the insured institutions. The ratio runs at values beneath unity, ranging from 0.72 to 0.93. When this ratio is close to 1.0, then the deposits equal the assets, indicating that the sole business of the institution is depository banking. As it turns out, nature seems to prefer that the industry sticks to the business of depository banking, having the higher ratio values during the quiet times, pre-1972. The modern trend is ever-decreasing historical values, this ratio at 0.717 in 1994.

The next chart introduces the final pair of ratios, the total deposit to interest-bearing liabilities, TD/IBL, and the interest-bearing asset to interest-bearing liability ratio, IBA/IBL. These ratios, with nearly identical levels and movements over time, reveal a critical interrelation which may prove key to operations.

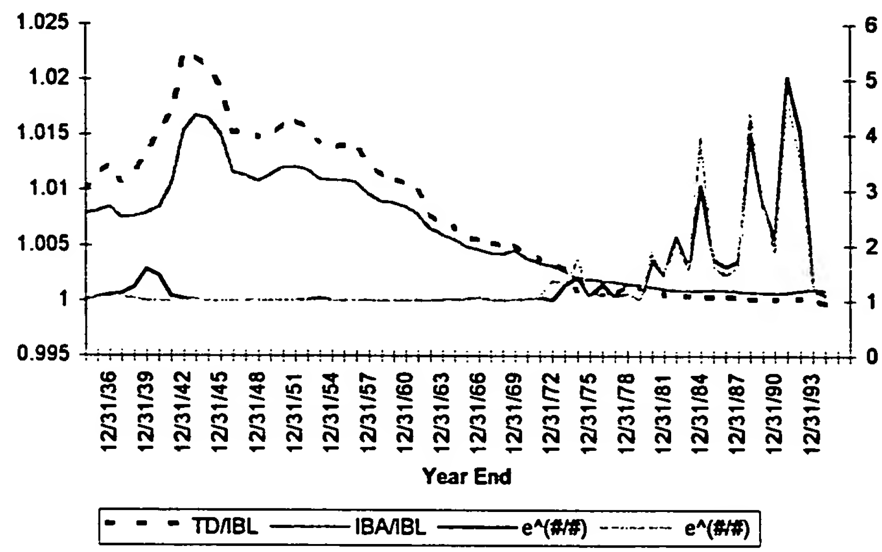
Ratio of Total Deposits to Interest-Bearing Assets and Liabilities plus Interest-Bearing Assets to Liabilities, 1934-1994



Observe that TD/IBL stays above the IBA/IBL ratio throughout 1934 to 1972, when it abruptly darts underneath it, never to again return above it. Notice, too, how the pending unsoundness of the banking industry, which seemed to spring at once, actually only slowly negotiated through the cross-over of the ratio, 1969 to 1979. The fact that the relation never rights itself, indeed grows more disparate in recent years, makes future calm impeachable. Note that TD/IBA, the two numerators of TD/IBL and IBA/IBL, shows the slow degradation, and switch of threshold, over history. Ranging 1.0 to 1.5 from 1934-73, TD/IBA suddenly dropped to 0.86 in1974, never recovering. Year-end 1994, an historic low value of 0.806.

From the above charts, the breadth of wide, safe ratios have gone the way of the gentleman banker, with the "three-six-three' motto (pay deposits 3%, lend at 6%, tennis at 3PM) but an antiquated expression. The following chart amply summarizes this section's findings. The operating ratios are overlaid with the ratios of total deposit closings to total insured deposits (e^#/#), and total deposit closings to insured assets. There is a dramatic and direct correspondence between these operating ratios and the rise in deposit default.

Historical Relation of Ratios to Deposit Closings



The final charts have depicted the historical relation between operating ratios and deposit closings, conveying the actual balance between unsettling discoveries and hopeful inquiries. The critical operating ratios converge in clear and definitive manners, compressing and crossing-over. The deposit loss proportions jump up, in this, the modern financial era. The evidence suggests the post-1972 peril is not past.

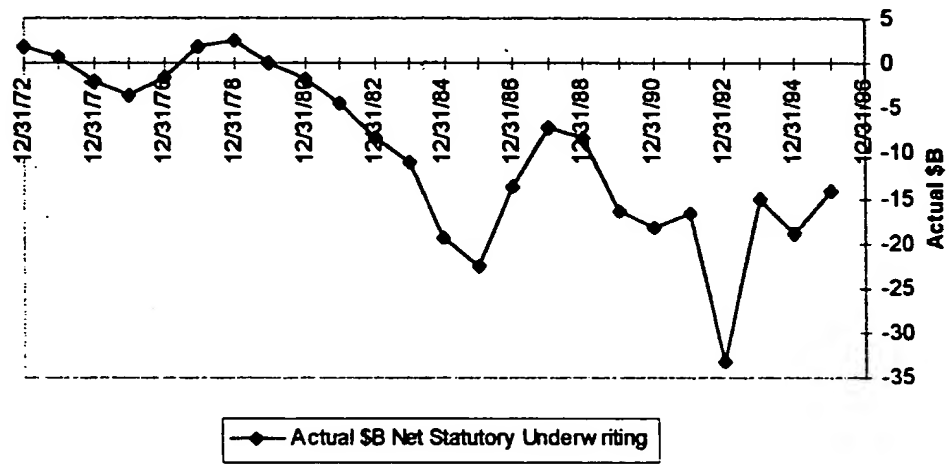
Arguably, a shake-out of the commercial depository institutions has run its course, with the losses over 1980 to 1992 having been 'survival' losses, associated with the competitive tightening of margins, real-estate losses, mergers, etc. , but otherwise non-recurring. In the last few years, total insured deposit closings have dropped to record low amounts. For 1996, insured deposits closed totaled under just $200M.

With the BIF presently standing at $27B, the FDIC, at banker insistence, has drastically reduced assessment rates for sound institutions to only $0.04 per $100 insured deposits, consequently generating only $0.07B assessment income for 1996, after about $6B for each of 1993 and 1994. In this period of low interest rates and tight spreads, the drag of the high fees was a profit killer. Now, the banks report record earnings, no longer paying 10 to 30 b.p. to the FDIC. Although the BIF earns another $1.3B in interest income on the fund per annum, no new surplus is accruing t BIF after FDIC expenses, with the provision for insured losses a negative $325M for 1996. The FDIC says it is confident, with the BIF balance as a percentage of total insured deposits exceeding its targeted ratio of 1.25 since 1994. How confident are you?

Part B: Catastrophe Losses and the Property & Casualty Insurance Industry

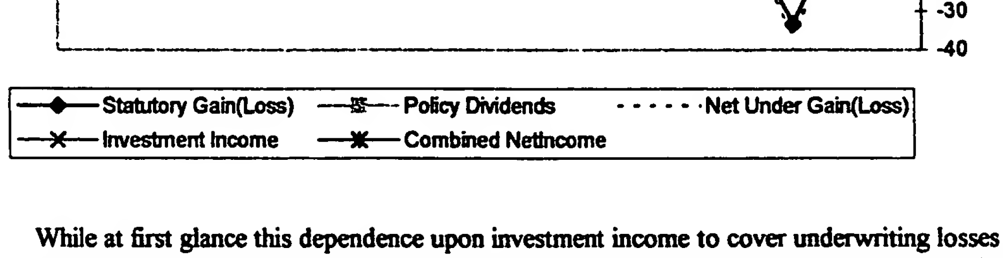
The Property & Casualty insurance industry reveals a primary, key, but very disturbing fact: for the last twenty years, their business of underwriting is a losing operation. The chart below shows, that on a net statutory underwriting basis, the industry has consistently posted l sses on its actual underwriting activity.

Actual $B Net Statutory Underwriting



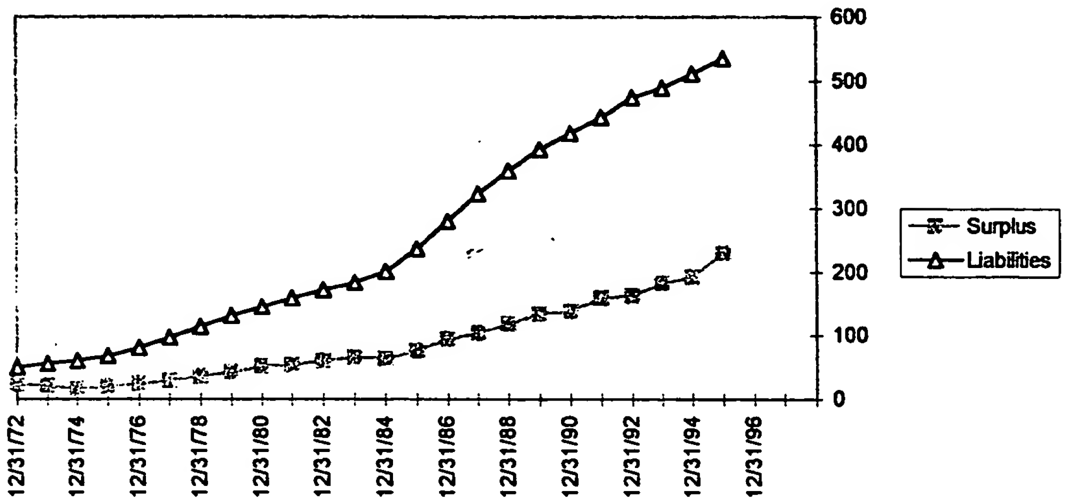
Nonetheless, the property and casualty industry consistently pays shareholders a positive return. How is this possible? The answer lies in the consolidated financial activity of insurance companies. As the next chart shows, the overall solvency of this industry is deeply indebted to the profound growth of investment markets during the modern era. It is only because of strong gains in the industry's investment income that deteriorating underwriting results are offset at final balance in consolidated financial statements.

Property & Casualty Operating Results



While at first glance this dependence upon investment income to cover underwriting losses may not seem consequential, in fact, it has had a profound impact upon the fundamental soundness of the industry as it has developed since the early 1970's. The chart below shows the significant lagging of surplus to liabilities in the pace of the industry's growth, financial returns included. Consider the gap in percent and $ magnitude.

Actual $, P/C Insurer's Surplus, Liabilities, 1972-1996



Thus, like the FDIC mis-adjusting the build-up of the BIF balance during the late 1960's into the early 1980's, the addition of new surplus, at rates commensurate with the growth of coverage to compound with previous reserves, was by-passed. Instead, the strong growth in pre-existing surplus, due to the investment market, obfuscated the fact that new surplus lagged well behind the pace of additional liabilities.

In both cases, a reliance upon historical ratios, maintaining surplus balances in line with levels of previous sufficiency, can overlook the risk as underlying liabilities transform, grow or concentrate. Hence, as bank operational ratios narrowed from the 1970's onward, the risk of bank closure greatly increased. The low interest rates and inflation prevailing today, with high growth and earnings, are reasons for this low loss.

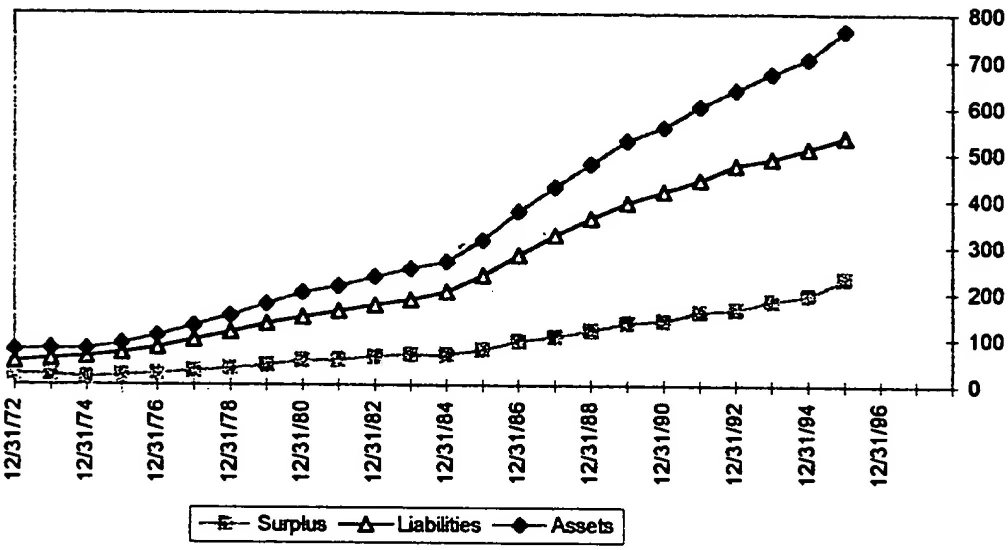
Similarly, P&C insurers face an increased risk of substantial losses to their operational returns. Though surplus to liability ratios, on an actual dollar basis, today nearly mirror those of the early 1970's, when a statutory underwriting gain was last recorded, the underlying risk of liabilities required ratios well beneath those appropriate today. Further industry operating ratios are found elsewhere, i.e. with A.M. Best.

The rise of inflation and interest rates began in the 1970's, with different impacts however, on the depository banking and property/casualty industries. For the bankers, this caused disintermediation, the loss of deposit base to other investment alternatives, as the bankers can not pay sky-rocketing short-term rates with lending rates locked in over the mid-term. In contrast, the rise in rates provided an opportunity, if only one-directional, to the P&C underwriters. This opportunity legitimated "cash-flow underwriting" practices.

Presuming a static proportion of losses to policy underwriting, the sharp rising rates presented insurers with an easy opportunity to expand short-term profitability by simply increasing the amount of insurance underwritten, investing these premiums in the higher-earnings of the capital markets. Assuming the ratio of losses to underwriting is fixed at a level beneath the return of investment-grade, profits are assured.

Consequently, new policies were underwritten at frenetic speed, without much regard for certain risk concentration in lines and treasury, as inflation kept rates high, with a return to low rate environments unlikely. To date, the publicly traded P&C insurers post regular dividends each and every year. But an over-extension of commitments int real estat and boom economies during the 1980's, plus the newer occurrences of catastrophic losses due to concentrations in vulnerable geographic areas, costs the industry billions of dollars each year. The chart below gives a sense to the dynamics of this liability-based growth.

Actual $B, P&C Insurer' Surplus, Assets, Liabilities, 1972-1996



Cash-flow underwriting is not the sole cause of the industry's statutory results. If it were alone, the dangers to P&C insurers would be effectively contained, since this practice has received attention and review in the industry. As long as liabilities are controlled and the favorable interest/inflation rates continue, profit at the consolidated balance sheet could continue. But a recent danger has emerged, an omnipresent uncertainty.

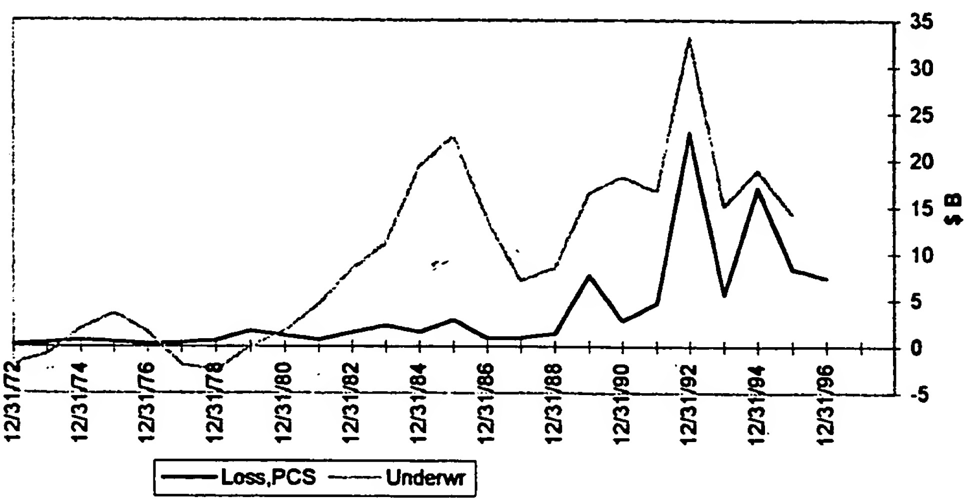
The risk of insured losses due to catastrophes. This risk had been moderate during the 1950's and 1960's, benefiting from a low bend in the natural cycle. Statistics indicate, on average, 1.2 significant hurricanes hit U.S. shorelines per year, yet on a decade basis since 1900, the annualized frequency ranges only from 0.80 to 1.45. But four factors have assumed startling potentials for loss and industry insolvency.

First, the liability-lead pursuit of cash-flow profits came at the expense of sound risk diversification. Insurers, in courting new policies relaxed concerns for certain aspects of insurance management. Second, the industry and its authority, NAIC, divided by state licensing and operational constraints, faced concentration in the regional exposures by individual insurance companies. Third, the 1970's political mandates, capping premiums and premium increases, requiring insurers to extend insurance to the public, forced insurers to supply insurance increasingly on terms which did not reflect full payments to the risks they bear. Fourth, the public being served, the politic which affirms their "rights" to insurance, and the nation's tax-payers who are all effected by these issues, has alarmingly grown in states and counties which are known catastrophe zones.

The demographic developments are profound. Published facts: By the year 2000, 75 percent of all Americans will live within ten miles of a coast; Between 1970 and 1994, the population in Florida, the most catastrophe-prone state, increased 95 percent, and by the year 2010, more than 70 million people will live in hurricane zones; The pace of insured property in vulnerable coastal locations is similarly astounding, as the total value of insured coastal property exposures increased more than 80 percent between 1988 and 1995.

Consequently, though the occurrences of catastrophes has remained roughly stable over the last fifty years, whereas previously these occurred in areas with little insured property, now property has been disproportionately concentrated there. As a result, the magnitude of insured losses to catastrophes has markedly increased, although the proportion of catastrophe premiums within the overall property/casualty industry has inversely declined from about one-quarter of premiums to about one-fifth of premiums. Given the mandated nature of premium rates and coverage, this contraction in premiums suggests a homeostasis beneath this portion, for this era when the exposure of insurers to catastrophic losses has clearly escalated.

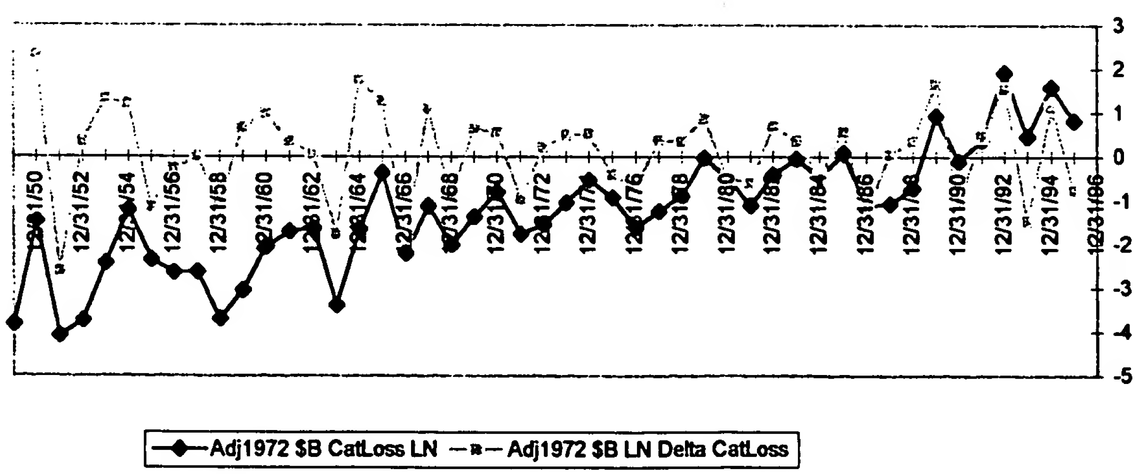
Actual $B, P&C Statut ry Underwriting Loss and Catastrophe Loss, 1972-1996



The above chart shows that, whereas previously the catastrophe losses had little impact on the statutory underwriting results, in the years since the late 1980's, it is the catastrophe losses which have had a devastating impact on operating results. It is important to realize that in these recent years, the occurrences of catastrophes have not appreciably changed, but rather that the configuration of insurance liabilities has changed. Regrettably, the demographic growth underlying this vulnerability continue to increase each year.

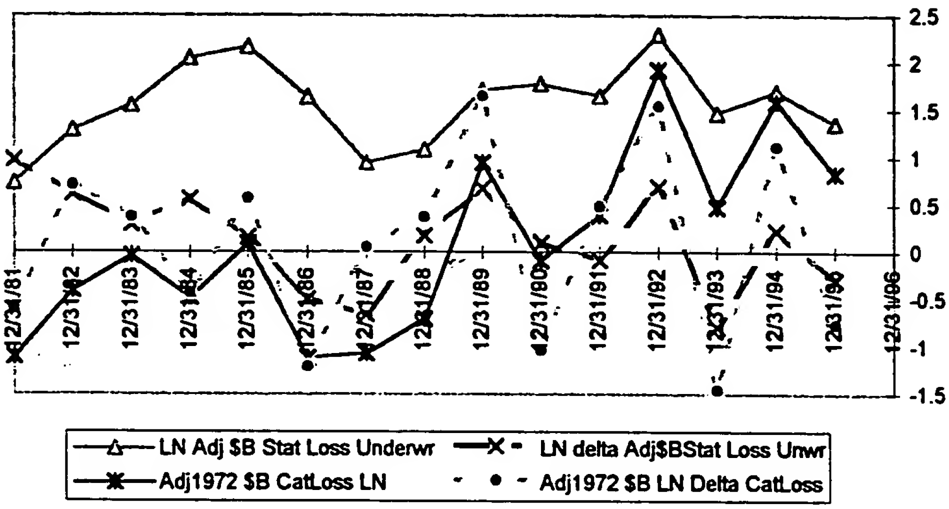
On a long-term perspective, the adjusted 1972 dollar loss amounts to catastrophes have been steadily increasing throughout their historic coverage, 1949 to present, yet, the recent acceleration is also manifest. The chart below documents this trend, showing the natural log values of losses and the log change.

LN Values for Insured Catastrophe Losses, 1972 Adj$, 1949-1996



One notices how the change (delta) in statutory underwriting losses had low volatility for two decades, later-1960's to the late 1980's. The delta cat loss after 1988 appears to have volatility similar to the period, mid- to later-1960's. The highest volatility years are actually to be found in the earlier periods of the data. The ascension of catastrophe losses, as the pivotal cause of the industry's statutory underwriting losses is particularly evident in the chart below, compare the Adj1972 $B LN lines, CatLoss to StatLoss. Notice how the catastrophe losses (its delta - dotted line) pull around the change in statutory losses after 1987. Also, note how the statutory l sses have lost m st of their non-catastrophe component after 1992 (LN $).

LN 1972 Adj$ Values f r Statut ry and Catastrophe L sses, 1981-1996



The occurrence of Hurricane Andrew in 1992 redefined the danger that catastrophe losses pose to the P&C industry. Indeed, those losses forced the industry to post a net operating loss above the substantial $33.7 billion earned from investment income. Moreover, it rattled the industry by revealing just how thin and vaporous the margin of solvency is within the chain of insurers and reinsurers, and how disaster could strike.

Presently, the capital of catastrophe reinsurers totals under $15 billion. This sum would be wiped out should a mega catastrophe strike a highly insured area such as Los Angeles, Dade County or New York City. The Probable Maximum Loss of a major strike to Los Angeles could exceed $50 billion in catastrophe loss, with another $50 billion in associated losses to insureds likely. Since Hurricane Andrew, the pool of reinsurers has not recouped from its devastating losses, and new capital has not entered this arena in force.

The contemporaneous analytic technology to assess and price these risks widely missed the mark, and as a result, a justifiable uncertainty now prevails, making those who underwrite these risks to demand a high risk charge. For instance, as reported by Applied Insurance Research, before 1992, a $10 billion hurricane loss was estimated as a one in twenty occurrence, while a $14 billion loss was given a once in fifty year chance. Hurricane Andrew caused insured catastrophe losses of $16 billion, and since 1990, every year has catastrophe losses of greater than $5 billion, hitting peaks of $23 billion in 1992 and $17 billion in 1994.

The technology used to assess high-end risks was defined around the calculation of the Probable Maximum Loss (PML). This technology bears more than passing resemblance to Value-at-Risk, the 'new' risk evaluation technology required of commercial and investment banks. The PML method begins by summing up an insurer's exposed premiums in predefined catastrophe zones (like VaR "buckets"), before applying multipliers (weightings) to factor likely sums for loss amounts. Risk and the pricing of risk then proceed from these PML numbers, with reinsurers quoting for coverage on layers above or below the PML.

Three factors contributed to the failure of PML in accurately assessing risk. During the twenty year period, 1968 to 1988, catastrophes, and particularly hurricanes, occurred at levels well under the average rate for the overall twentieth century. This had an effect to hide the growth of risk concentration in catastrophe prone zones. Second, the PML method was based upon finding values equating to 95 percent confidence levels. Catastrophe losses, as a variable, have a remarkably long tail, being lognormal, yet proportional or excess of loss reinsurance was often written without upper limits. Catastrophe reinsurance contracts would require careful modeling and implementation to insure the extreme hidden loss potential. But severe, rare events are extremely difficult to budget, with uncertain swings in year-end results expected.

Theoretical Engineering, Associated Mathematics and Numerical Techniques

Respective Single State Variable, Theta, Underlying Each Type of Loss

An investment or derivative security might be modeled by using a single underlying state variable for the depository losses or for the insured catastrophe losses. A state variable, theta, is held to follow an independent Markov stochastic process: $d\theta/\theta = m\ dt + s\ dz$. This states that the future value of θ depends on the known present values under continuous pricing. As Wiener process, dz is related to dt: $\Delta z = \varepsilon\sqrt{\Delta t}$.

A theta variable depends solely on itself and time to define an expected drift and volatility, which it redefines throughout the course of its life. Thus, $d\theta = m(\theta,t)\ dt + s(\theta,t)\ dz$. For methods drawing from standard normal distributions, i.e. Black-Scholes, geometric Brownian or Monte Carlo process, the log value of the change of theta over time and/or the log of theta at exercise should have this distribution.

This theta, single state approach has been selected here, since the target variables are not the prices of traded securities. For the insured deposit variable, the "deposits closed" and the "deposit loss" are candidates for that industry's theta. For the insured catastrophe dollar risk, they are the "catastrophe loss" and net "statutory underwriting loss". Variables can be created from divers theta, called "multiple state".

Towards creating a tradable instrument, assign the function, f, as the price of a security dependent only on θ and time. For instance, for θ(banking) and θ(cat), let f(b) and f(c) be the respective price of a derivative security with payoff equal to a functional mapping of θb and θc into the future. Let the processes of f(b) and fc be defined via Ito's lemma, where $df/f = \mu\ dt + \sigma\ dz$. This would stand for any f(theta).

On a continuous time basis, the change in the price of a security dependent on the deposit losses is $df(b) = \mu b\ fb\ dt + \sigma b\ fb\ dz$. An instantaneously riskless portfolio can be created from a combination of related f(bi), such that $(\mu 1 - r)/\sigma 1 = (\mu 2 - r)/\sigma 2 = \lambda$. Thus, for any f, being the price of a security dependent on only θ and time, with $df = \mu\ f\ dt + \sigma\ f\ dz$, there is the parameter lambda, $\lambda = (\mu - r)/\sigma$, which is dependent on θ and time, but not on the security f. This is the market price of risk of θ.

The variable μ is the expected return from f. The expected drift, mu, equals μf. Sigma, σf, is the volatility of f(θ), and is either positively ($df/d\theta>0$) or negatively related to θ. If negative, volatility = $-\sigma$, and df = mu dt + $(-\sigma)$ f (-dz). The variance is [(σ^2)(f^2)] and dz is over an independent interval, dt = (T-t).

Using Ito's lemma, the parameter μ is set as μ* f = df/dt + m θ df/dθ + ½ s^2 θ^2 d2f/dθ2. The parameter Sigma is set as σ * f = s θ df/dθ. This results in a structure mirroring the Black-Scholes differential equation, df/dt + θ df/dθ (m - λ s) + ½ s^2 θ^2 d2f/dθ2 = r f , with S replaced by θ. This equation can be solved by setting the drift of θ equal to (m - λ s), and discounting expected payoffs at the risk-free interest rate. Thus, under risk-neutral valuation, the drift of θ is reduced from m, to (m - λ s).

Towards the construction of a valuation lattice, introduce the notions of delta, δ = e^(σ*sqrt(Δt)), and of mu = [2*e^(r*Δt)]/[δ + δ^-1]. Hence, σ = ln(δ)/(sqrt Δt). Next, set values for Δt, sigma, r(t) and S (if modeling a stock), calculate nodes of S at S(tk) = [mu^k]*[δ^w(k(w))]*[So]. Substituting θ for S affords an underlying random walk of w(k(w)), such that if w=(-1,-1,1...), S(t(3)) = [mu^3]*[δ^(-1)]*[So].

In lognormal world, this relation becomes: ln S(tn) = [n * ln mu] + [w(n(w)) * ln δ] + [ln So]. This results in the equalities: ln δ = σ*(sqrt Δt) and dt = T/n. By substitution and by letting k = n, such that tn = T, this form becomes: ln S(tk) = [n * ln mu] + [w(n(w)) * (σ*sqrt Δt)]/[sqrt n] + [ln So]. More simply, E(ln S) = ln mu + ln So. The Var(ln S) = (ln δ)^2, and the Volatility of S = (ln δ)/sqrt Δt. For a pathing tree, the typical node value mechanic, S(tn) = [mu^n]*[δ^w(n(w))]*[So], using logarithmic transform, results in the node mechanic, ln S(tn) = [n * ln mu] + [w(n(w)) * (ln δ)] + [ln So].

By the Central Limit Theorem, the term, $w(n(w))/(\text{sqrt } n)$ is shown to exhibit strong convergence to the standard normal $N(0,1)$. The term $[n^* \ln mu]$ shows weak convergence to $[(r - ½ \sigma\char`^2)^*T]$, hence its implementation is limited to discrete methods. The term $[\ln S(T)]$ is distributed as $[(r - ½ \sigma\char`^2)^*T] + [N^*\sigma^*(\text{sqrt } \Delta t)] + [\ln So]$. Non-log, $[S(T)]$ is distributed as: $[So * e\char`^((r- ½ *\sigma\char`^2)^*T) + (N^*\sigma^*(\text{sqrt } \Delta t)]$.

Returning to continuous valuation of a derivative security based upon the state variable theta, consider the European call option, with realizable cashflow only at T, value today of C, with the functional mapping, f(So) = max [S(T) - K, 0], where K is strike price and S is held substitutable by theta. Using weak convergence, today's value for C(S), based on S at T, can be derived over the normal distribution:

$$C(S) = [So * \phi\{(rT+\ln(So/K))/(\sigma^*(\text{sqrt } dt)) + (1/2\ \sigma^*(\text{sqrt } dt))\}] - [Ke\char`^(-rT) * \phi\{(rT+\ln(So/K))/(\sigma^*(\text{sqrt } dt)) - (1/2\ \sigma^*(\text{sqrt } dt))\}]$$

This is the Black-Scholes Formula, representing $C = e\char`^(-rT) * E(m)[S(T) - K]$, with E(m) being the expected value under risk-neutral conditions. Similarly, for any function, f, valuing a derivative security based on theta that pays off f(T) at time T, the expected risk-neutral value is $f = e\char`^(-rT) * E(m)(f(T))$. This requires setting the growth rate of the underlying to $[m - \lambda^*\sigma]$, rather than as m alone.

The final product of risk-neutral valuation is the statement that today's value, f(0), of a derivative security paying off f(T) at time T, is equivalent to the risk-free discount over period (0,T) of its expected risk-neutral future pay-out. This narrow evaluation is valid for f only over the continuous segment (0,T), with determinable values of F(0) and F(T). Lattice arguments which sub-divide this segment are weakened if their Δt-parameters, i.e. $\Delta t = (T-t)$ with $0<t<T$, are modeled using analytic values from (0,T) data sets.

Moreover, the Black-Scholes argument, or any methodology which relies on convergence to a lognormal distribution for its valuation or simulation, is strictly consistent only for European-style derivatives, that is, having exercise only at T, but not continuously throughout the segment (0,T). Also, it assumes the security can gain or lose value during (0,T), with the price of the security always non-negative.

The payoffs of our θb and θc securities can be European, if these stem from the single terminal condition of theta at T: the selected theta variables are annual aggregates, they begin each year at $\theta=0$ and end the year at T, $\theta \geq 0$, European (0,T) events. For rigorous risk-neutral valuation, strict conformity can only be assigned under a European-style (0,T) segment, variable and security. In the words of Hull:

"Equations (12.14) [ed.: $f = e\char`^(-r(T-t)) * E(m)\{f(T)\}$; f=f(0), (T-t)=(T-0)] and (12.15) [ed.: $f = E(m)\{e\char`^(-r(T-t) * f(T)\}$], with r=average risk-free rate over (T-t)] are true when the payoff, f(T), is some function of the paths followed by the underlying variables as well as when the payoff depends only on the final values of the variables. In the former situation, f is termed a *history-dependent derivative security*."

The premises of the valuation functions and the underlying theta variables, meet this criterion of a history-dependent derivative security. It defends implementation of risk-neutral and stochastic methods on the problem of deposited and insured losses, assuming the variables conform. Importantly, these theta can only be substituted for S for continuous trading after adjusting for the conditions that θ at $T = \Sigma\theta i$, each θi occurring and aggregating discretely over (0,T). For continuous trading, full data on the annual path of our thetas over (0,T) are required and are available. If the security is not trading (0,T), this is not necessary.

Substituting in the valuation function for a security or derivative instrument dependent only on theta and time, stating V(0,T) as the identity of $\theta(0,T)$, $V(t,0)=V(t)= Vo * e\char`^[(r- ½ \sigma\char`^2)t + (N^*\sigma^*(\text{sqrt } t)]$. This requires only that the natural log of the change in theta, hence, in V, has a characteristic, normal distribution. However, if this change in theta does not have a true mean of zero, but shows some regular non-zero value, even if it be small, ie. 0.1, 0.05, etc., then a mu-based, risk-neutralized, implementation is required. Thereby, the reader is returned to the opening of this section, where those methods are discussed.

Theta-Based Swap

A swap is constructed as two sides, for the bankers modeled on deposit losses as theta(b) and f(b) as the valuation function, while for the property/casualty insurers, it is based on their loss state variable, theta(pc) a valuation function of f(pc). The value of the swap to the payer of the deposit losses, f(b), assuming the swap of all year-end aggregate losses, is V = e^(-rT) * {E(m)[f(pc)(T) - f(b)(T)]}.

Though a swap is composed of two sides, its value, V, is a single function. Thus, V is a single derivative instrument modeled by the expectation of the two functions, each respective of its own single theta variable. Consequently, this function, V, can be also modeled as a security dependent on unrelated state variables, theta(i). Each theta(i) follows a stochastic process of form $d\theta/\theta = mi*dt + si*dzi$, with mi and si the expected growth and volatility rates, dzi being Weiner processes, then substituting V for f, the total loss swap, V, has the form, $dV/V = \mu\, dt + \Sigma[si*dzi]$, with μ being the expected return of the swap. The component risk of the return due to the theta(i), $\Sigma[si*dzi]$ must be adjusted if the theta(i) are correlated.

Merton's Put-Option Model for Deposit Guarantees, based on Deposits and Assets

In closing this technical excursion, it may prove useful to consider the seminal conceptualization by Robert Merton, 1977, on deposit loss guarantees. Understanding that the payout on a guarantee is the payment of the insured loss, minus any recovery on the underwritten (collateralizing) assets, Merton argues that a deposit insurance guarantee for a period term is isomorphic to a European put option. This assumes the valuation and settlement dates coincide exactly end of term, with no exercise except at terminal T.

The value of the guarantee to the insured banks, can be stated as G(0) = max (0, B-V), where the V=Bank Assets and B=Bank Deposits (Insured); i.e. B=K, V=S in typical option symbols. The Put (0,T) is modeled by Black-Scholes technique, using the normal distribution, $G(T) = Be^{\wedge}(-rT)\phi(X2) - V\phi(X1)$; with $X1 = [\ln(B/V) - (r+(\sigma^{\wedge}2/2))(T)]/[\sigma*\mathrm{sqrt}(T)]$ and $X2=X1 + (\sigma*\mathrm{sqrt}(T))$. Assumed is lognormal distribution of the bank assets, or minimally, a normal distribution of dV at T, for a security exercisable only at T.

The cost of riskless guarantee, % of amount covered: G(T)/Be^(-rT) = 1 - e^(-[R(T)-r]*T), where R(T) is promised yield over T and r is risk-free for T. The Cost of Guarantee, per $ of insured deposits, G(T)/D: $g(d,tau) = \phi(h2) - ((1/d)*\phi(h1))$; h1 = [ln(d) - tau/2]/sqrt(tau) and h2 = h1 + sqrt(tau), with d= D/V (current deposit to asset ratio) and tau = $(\sigma^{\wedge}2)*T$ (variance in ln change in value of assets over term).

Merton argues that these Black-Scholes techniques can be applied to the pricing of corporate liabilities in general: applying to Catastrophe Insurance: V = Cat Reserves+Surplus and B = Promised Coverage, using industry numbers for surplus, losses and probability distributions. Applied to bank closings, recovery rates are key to valuation: Deposit Closings vs. Loss after Recoveries. Note that the Merton instrument transfers asset recovery to the guarantor, hence is a complicated, collateralized security.

Reinsurance

Insurance transfers risk. Risk implies to insurers uncertainty accompanied by the possibility of loss. Insurers provide two functions, the transfer and pooling of risk. By pooling together numerous individual risks, insurers gain the advantage of the law of large numbers, whereby as the sample size increases, the experience of average loss per exposure converges towards the expected distribution. For actuarial sciences, statistical experience of frequency and severity of loss, homogeneity of individual risks, and usable forms of loss distributions, are central requirements for the effective underwriting of insurance.

Reinsurance is the insurance of insurers by other insurers. It can be done on treaty (portfolio) or on per risk basis. It may apportion losses by pro rata (facultative and surplus share) or on an excess of loss basis (i.e. 400 XS 100). Reciprocal and syndicated arrangements have been used for high capacity risks.

Actuarial Sciences

Regarding the variables under investigation in this study, let this section integrate certain insurance concepts and actuarial sciences with this topic. On a per bank deposit insurance basis, the distribution is binomial, either the bank deposit are solvent, or the bank is closed insolvent. This default is a single loss, one occurrence, notching one in frequency. On a collective basis, such as reinsuring the total insured banks, for instance through treaty with the FDIC, each loss in the aggregation counts, one occurrence = one frequency.

Collective risk models assume a random process that generates losses for the insured portfolio. The aggregate insured loss for the portfolio, S, in period (0,T) is: $S = X1+X2+\ldots Xn$, where the Xi are the individual losses, X1 being the first loss, with N losses in (0,T). The losses, Xi, are independent, identically distributed random variables. $E(S) = E[E(S|N)] = E[P(0,T)N] = P(0,T)\,E[N]$, P= common d.f. of Xi's (0,T).

Generally, mean frequency is the number of occurrences divided by the exposure to risk basis, i.e.: # closed banks/#insured banks. Mean severity is sum of total losses from all occurrences, divided by the number of occurrences. Together, [frequency x severity] = [pure premium/exposure to risk basis]. Pure premium (p.p.) is relative to the total dollar amount of insured base, i.e. [$ total closings/$ total deposits]. Premium = (p.p. x exposure) + expenses + risk charge. Risk charge is the payment to surplus owed to risk fluctuations due to miscalculation or to non-expected behavior in the values of the modeled variables.

The actuarial sciences are highly developed with respect to the modeling of variable distributions, as well as the incorporation and implementation of historical data sets. These advanced skills are important contributions of the insurance industry to the greater financial engineering community and financial industry. Capital markets valuation and analysis relies on rigorous estimations of the variables' present characteristics.

A normal distribution is the "bell-shaped" form, with mass centered toward the middle of the range, and lowest mass at both ends of the range, characterized as $N(\mu,\sigma^{\wedge}2)$, with inflection, $x=\mu-\sigma$. A variable is lognormal, if the log of the variable has normal distribution. Visually, lognormal variables, without log conversion, show distributions that spike up early in the range and quickly recede, but decay over a long ending tail. Hurricane loss has been documented as lognormal (see ogive and histogram in Hogg, Klugman).

Keeping X as $N(\mu,\sigma^{\wedge}2)$, setting $x = \ln y$, Y is a positive-value random variable with lognormal p.d.f.: $g(y) = [(1/\sqrt{(2\pi\sigma)}]\exp\{[-((\ln y - \mu)^{\wedge}2)/(2(\sigma^{\wedge}2))] \times [1/y]\}$, $0<y<\infty$. The expectation of Y is $E(Y) = \mu(Y) =$ m.g.f. of X at (t=1): $(Mx1)= \exp[\mu + (\sigma^{\wedge}2/2)]$. With $E(Y^{\wedge}2) = \exp[2\mu + 2(\sigma^{\wedge}2)]$, the variance of Y is $[\sigma(Y)]^{\wedge}2 =$ m.g.f. of X at (t=2): $(Mx2) = E(Y^{\wedge}2) - \exp[2\mu + (\sigma^{\wedge}2)] = \exp[(2\mu + \sigma)^{\wedge}2] \times \{\exp[\sigma^{\wedge}2] - 1\}$.

Joint distributions and their associated mathematics are necessary and useful in the modeling of two or more data sets, periods within data sets, or related or unrelated variables. For the modeling of joint random variables, the values for their correlation coefficient, covariance and conditional distributions need be determined. The terms of independence, dependence and mutual independence also require consideration.

In the words of Rodermund, quoted in Herzog, "the concept of credibility has been the casualty actuaries' most important and enduring contribution to casualty actuarial science". Credibility theory handles how two or more data sets, periods within data sets, or variables described by data sets, can and should be mixed. The classical paradigm is based on empirical frequency, while Bayesian has subjective probabilities.

Brownian Motion and Pinned Simulation

Brownian motion defines the change in the value of a variable as related to the variable's initial value and characteristic deviation, as well as to distinct random perturbations over independent intervals. It is at heart, a discrete process that approaches continuous form when the intervals are small and uncorrelated. Pinned simulation fixes an initial and terminal value for the variable, developing the value path in between.

As an expression of theta with respect to time and to Brownian motion, the life of theta over (0,T) and projected in simulation, can be related in derivational form, $\theta(t,B) = \theta o * e^{[(r - \frac{1}{2}\sigma^2)t + \sigma * B]}$. This equation values without preference to risk. It renders risk-neutral results by removing all representation of a mean for theta. This holds regardless whether theta is also held as a variable respective of Brownian motion.

However, for geometric Brownian motion, the theta variable must be lognormal in functionality. To implement this when modeling theta, respective of time only, $\theta(t) = \theta o * e^{[(r - \frac{1}{2}\sigma^2)t + \sigma * sqrt(t)]}$. The weak convergence by $\Delta\theta(t)$, requires only that the natural log of the change in theta shows a normalized distribution and characteristic variance (not necessarily $\sigma^2=1$). This is a weaker requirement on theta.

Allowing the notation, $\theta(t,B) = \theta o * e^{[(r - \frac{1}{2}\sigma^2)t + \sigma * B]}$, the change in θ, $\Delta\theta(t,B)$, measured at the terminal values (0,T), with t=T-0, can be derived as: $d\theta = \theta(B)dB + [\theta(t) + \frac{1}{2}\theta(BB)]dt$. The partial derivative input parameters are: $\theta(B) = d/dB$ of $\theta(t,B) = \sigma * \theta$; $\theta(BB) = \sigma^2 * \theta$; and $\theta(t) = [r - \frac{1}{2}\sigma^2] * \theta$. This computes as $d\theta = \sigma * \theta * dB + [(r - \frac{1}{2}\sigma^2)\theta + (\frac{1}{2}\sigma^2)\theta]dt$. In discrete form, $\Delta\theta = \sigma * \theta * \Delta B + r * \theta * \Delta\theta$. For normal theta variables, respective only to time and theta, $\theta(t) = \theta o * e^{[(r - \frac{1}{2}\sigma^2)t + (N * \sigma * (sqrt\ t)]}$.

Substituting in the valuation function for a security or derivative instrument dependent only on theta and time, stating V(0,T) as the identity of $\theta(0,T)$, $V(t,0)=V(t)= Vo * e^{[(r - \frac{1}{2}\sigma^2)t + (N * \sigma * (sqrt\ t)]}$. This also requires only that the natural log of the change in theta, hence, in V, has a characteristic, normal distribution. N represents a sampling off the standard normal, and is equivalent to the symbols already used, ε and ϕ.

Monte Carlo Simulation

Monte Carlo simulation is a discrete methodology that is based on large numbers of sampling sequences. The life of the security is subdivided into N intervals, each of length Δt. Using, s≡ volatility, and m≡ risk-neutral growth rate, of θ, for one underlying variable, $\Delta\theta = m * \theta * \Delta t + s * \theta * \varepsilon(\sqrt{\Delta t})$. Each simulation run has N drawings, one per Δt. For a multiple state θ: $\Delta\theta i = mi * \theta i * \Delta t + si * \theta i * \varepsilon i(\sqrt{\Delta t})$, with θi: $(1 \le i \le n)$. If the θi are correlated, one needs to incorporate the instantaneous correlation between the θi, ρik, and also between the εi, ρik. This requires that the εi are sampled from a multivariate standard normal distribution.

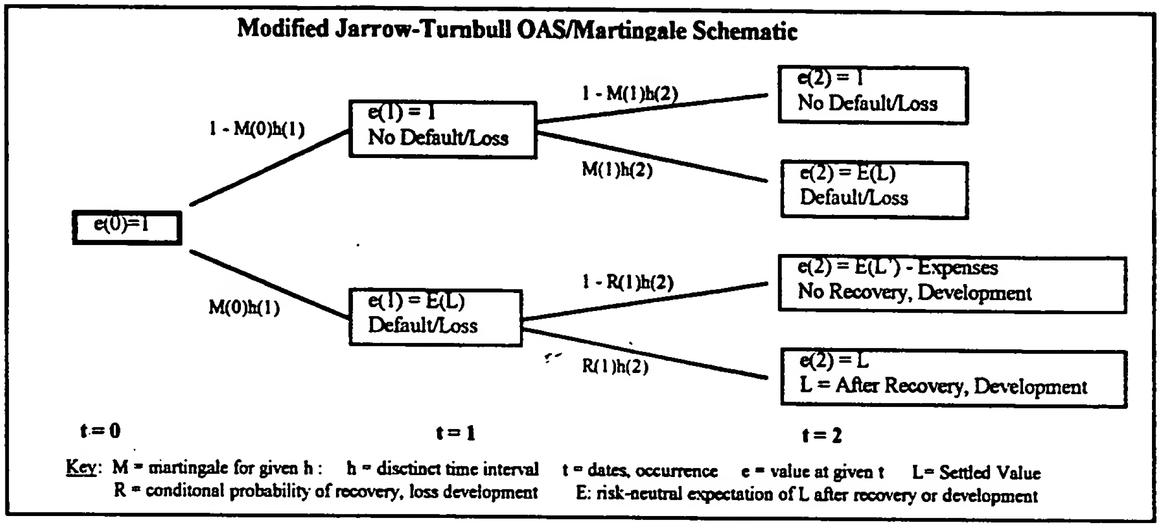
Martingale Lattice

Conditional probabilities, and measures of correlation, covariance and independence are addressed by the actuarial sciences, in joint loss distributions and in credibility theory. Conditional probabilities play a key factor within any multi-state theta variable, or lognormal derivative security, or combinations, with basis. The capital markets have their own conditional probability technology, the martingale, conditional over time.

A martingale is the conditional probability of an occurrence, such as default, over a time interval. It is utilized in credit analysis, i.e. to evaluate the valuation of a default-risky bond, relative to the bond market. In this mode, the martingale is applied relative to the credit spread between bonds, priced against the default risk-free yield curve. This mechanic uses the option-adjusted-spread, OAS, over a discrete Δt pathing lattice.

An OAS/martingale lattice is often used to price callable investment instruments, this isomorphic to an instantaneous guarantee against losses. In this mode, the Merton bank default guarantee, a put option, assumes exercise and settlement to occur together at T. However, loss development says this does not simultaneously occur. Further, for bank default, the recovery against assets is not realized at bank closing.

The OAS methodol gy itself, as well as any Black-Scholes style valuation, only affords rigorous pricing for highly liquid and capitalized classes of financial instruments. As the volatility smile shows, option value is not just a function of time, volatility, security value and strike, but also of liquidity. Liquidity impacts its theoretical price. For robust instruments, the contracts far from-the-money have characteristically higher implied volatility, being thinner there. For lesser exchange and OTC instruments, this is a random volatility.



Transactions and Instruments

The derivative instruments that have come to market for the catastrophe insurers, such as the CBOT contracts, have generally not met with success. Other initiatives have difficulty. In both cases, the relative smallness of this industry's capacity, only \$150B for \$15T of insured real estate, plus \$15B surplus in reinsurers, vs. say about \$5T in U.S. Treasuries, means that exchange trading faces liquidity blockades.

Also, there are few willing sellers of put contracts. There is demand on the part of insurers and reinsurers for further protective contracts from losses due to catastrophe. In addition to the Bermuda Commodities Exchange for cat options, recent efforts have focused on the issuance of catastrophe bonds and private placement trust units. These cat bonds are typically treaty XS, on behalf of a securitized insurer.

In fact, they look like project finance from a decade or two ago, using zero-coupon U.S. STRIPS to accrete to, and guarantee, par at the termination of the unit/bonds. The new issues, regrettably, are limited to just the largest insurer/reinsurers, and typically, provide only "short-tail", limit-loss, protection on the insured cat loss. Thus, these may not even provide these largest insurers with coverage ensuring solvency.

The trust units and securitized bonds meet obligations through using part of the issuance proceeds to buy the zero's, with the majority remainder placed as collateral for the operating fund. Municipal debt can be underwritten similarly. So far, these transactions provide capital on the order of \$100M to \$500M per, but just a handful of the assorted, alternative catastrophe risk instruments have been transacted successfully.

Credit derivatives provide a slightly more established market than those for cat risk. The estimates on the size of the global credit derivatives market range about \$40B, a nip of the credit-risky market. Deal size tends small, \$10M to \$50M, outside of the occasional \$100M+ issues of credit-linked notes and trust securities by a few top banks. Credit deals linked to default include asset and total return swaps and options. Generally, these are one-off transactions, bilateral to a bank underwriter. Some are high leverage, quick loss.

Other types of instruments suitable for cat risk or for deposit default come from the merchant banks and global finance and investment houses. These vehicles, drawing on vast global capital capacity, include:

standby letters of credit, which might allow one bank to guarantee on the deposits of another;
bank guarantees, available only from non-U.S. banks and finance institutions, t expand capacity;
forfaiting, the financing function on contractual receivables, such as premiums, for interim shortfall.

These instruments and vehicles are well established as markets and in contracts, jurisprudence and ICC code. These can be structured as non-transferable, assignable or bearer, term or on-demand, even, interest-bearing.

Statistical Evaluation of Variables

At the outset of reviewing the statistical data and distributions for deposits closed/losses and catastrophe losses/statutory underwriting losses, one cannot help but be struck by a balance in magnitudes between them, making perhaps even a swap of deposit closings versus catastrophe losses possible. For instance, from 1989-1995, the mean log adjusted-dollar deposit closings is virtually identical to that of catastrophe losses, 2.05 versus 2.06, with correlation between these two of about zero, actually minus 0.15.

On an aggregate basis, the total outstanding dollar value of insured property along coastline, \$3.5T, roughly splits the total of insured deposits (\$2.9T) and assets of insured banks (\$4.0T). Similarly, the actual-dollar value for statutory underwriting losses, approximates the mean actual-dollar value for deposit closings over the each of the modern periods tested.

Actual \$B	**μ Statutory Underwriting Loss**	**μ Deposit Closings**
1973-1995	10.1	10.7
1981-1995	15.2	15.7
1989-1995	19.0	19.5
max loss (year)	33.3 (1992)	53.8 (1991)
Note: Correlation of CatLoss to StatLoss:	1979-95: 0.563	1989-95: 0.663

In the parts that follow, the principal variables are the deposit closings and the catastrophe losses, with the deposit losses (closings minus recoveries) and the statutory underwriting losses relegated to a subordinated position. Deposit losses are a transform of the precursor variable, deposit closings. To the extent that an insured bank is closed, only then does the occasion to recover against the asset based occur. Further, if any proposed transaction basis is not to require or convey the actual transfer of notional value, then deposit closings, distinct of any recoveries or operational involvement, is the path to pursue.

The focus upon catastrophe losses rather than statutory underwriting losses clearly addresses a key and distinct value, whereas, the magnitude of underwriting losses is inherently tied to any variety of factors that impact the net results of P&C insurers. Also, to focus a transaction on the statutory net loss could create a disincentive for the industry to perform to the best of its ability.

Towards the consideration of the catastrophe losses and the deposit closings as theta variables, the following table shows the descriptive statistics for their change from year-to-year on the log of 1972-adjusted values. These values have not be scaled for growth in the total amounts insured over time.

Delta LN Adjusted-Dollar: Deposit Closings versus Catastrophe Losses

Period:	1934-95	1949-95	1973-1995		1981-1995		1989-1995	
	DepCl	CatLoss	DepCl	CatLoss	DepCl	CatLoss	DepCl	CatLoss
Mean	**0.0977**	**0.0956**	**0.0940**	**0.1031**	**-0.1859**	**0.0889**	**-0.6136**	**0.2195**
Median	0.192	0.346	-0.315	0.363	-0.468	0.363	-0.528	0.478
Std. Dev.	**1.471**	**1.042**	**1.643**	**0.843**	**1.066**	**0.976**	**1.139**	**1.293**
Variance	**2.163**	**1.086**	**2.700**	**0.710**	**1.137**	**0.953**	**1.297**	**1.673**
Kurtosis	0.61	-0.05	0.61	-0.64	0.70	-1.01	2.55	-2.18
Skew	0.096	-0.33	0.72	-0.06	-0.26	0.004	-0.20	-0.18
Correlation		**-0.122** (1949-95)						

For the longer periods, 1934-1995 and 1973-1995, one sees that these two variables post similar mean values, 0.09-0.10. The change of catastrophe losses show standard normal type values for σ and variance, each near to one, within tolerance of the small sample size involved. The deposit closings do not.

In the most recent periods, the change in the deposit closings show σ and variance just above one, but my small sample-size (N=7, N=15, N=23, etc.) standard normal Box-Muller sequences do not post values above one when mean is negative. The mean change of catastrophe losses have recently jumped well above the consistent values over the longer periods. This indicates that 1989-1995 appears aberrantly high.

Over the course of the modern sample period, one sees how the mean catastrophe losses have grown relative to deposit closings, such that in the most recent period, 1989-1995, these two variables now are about equal. The chart below shows the log values of the adjusted-dollar figures for these two variables. From this data, the catastrophe losses show small sample N(0,1) for the periods, 1973-1995 and 1981-1995.

LN Adjusted-Dollar: Deposit Closings versus Catastrophe Losses

Period:	1934-95	1949-95	1973-1995		1981-1995		1989-1995	
	DepCl	CatLoss	DepCl	CatLoss	DepCl	CatLoss	DepCl	CatLoss
Mean	**-2.500**	**-1.196**	**0.489**	**-0.240**	**1.117**	**0.081**	**0.858**	**0.866**
Median	-3.12	-1.12	0.84	-0.48	1.14	-0.04	1.55	0.83
Std. Dev.	**2.730**	**1.388**	**1.532**	**0.924**	**1.301**	**0.951**	**1.813**	**0.703**
Variance	**7.454**	**1.926**	**2.348**	**0.855**	**1.137**	**0.903**	**3.286**	**0.494**
Kurtosis	-1.00	-0.11	-0.55	0.16	0.42	-0.52	-1.76	-0.67
Skew	0.39	-0.06	-0.43	0.83	-0.87	0.49	-0.45	0.36
Correlation		**0.701** (1949-95)	1969-95: **0.605**		1979-95: **0.023**			**-0.148**

The statistical data support interest in transactions that are based upon catastrophe losses and deposit closings. Clearly, the catastrophe losses have fairly strong standard and lognormal character, which is suited to theta, single state, modeling. The deposit losses have long-term characteristic mean, σ and variance, but only the delta, change form year-to-year, values exhibit near standard normal character. Thus, deposit losses could be used in theta or multiple-θ transactions, assuming there is no interim trading (0,T).

A valid reason for using the variable, deposit closings, for the banking loss theta, lies in the fact that for its pre-modern history, the recovery after closing was been nil or trivial. However, over the last two decades, recovery has improved, with an average closed institution having some residual liquidation value. The table below shows this trend and relations between deposits closed and deposit losses. After recovery, the data, 1981-95, show losses roughly equals long-term deposits closed for deviation (1.4), variance (2.1).

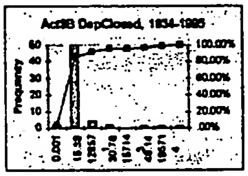
LN Adjusted-Dollar and Delta LN: Deposit Closings versus Deposit Losses

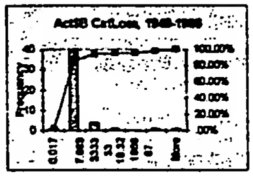
	LN Adjusted-Dollar				Delta LN			
Period:	1981-1995		1989-1995		1981-1995		1989-1995	
	DepCl	DepLoss	DepCl	DepLoss	DepCl	DepLoss	DepCl	DepLoss
Mean	1.117	0.750	0.858	0.399	-0.186	-0.228	-0.614	-0.688
Std. Dev.	1.301	1.428	1.813	1.961	1.066	1.237	1.139	1.330
Variance	1.137	2.039	3.286	3.845	1.137	1.531	1.297	1.769
Correlation								

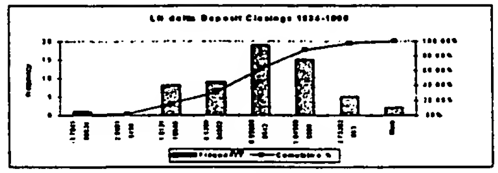
The above tables raise topics of data sets that appear to have Poisson characteristics ($\mu=\sigma=\text{Var}\approx 1$). These distributions, especially of rare, highly kurtotic and severe events may resemble jump functions. Here, the parameter lambda, λ, defines values ($\mu=\sigma=\text{Var}\approx\lambda$). For 1981-95, the log values for deposit closings are: $\mu=1.117$; $\sigma=1.301$; Var=1.137. For 1989-95, these values for cat loss are: $\mu=0.858$; $\sigma=0.703$; Var=0.494. Such periods mark the longer history, this latter being normal, due to the effects of pooling. It is noted, that as an insurer, the FDIC has the benefit of the full pool of insured banks, population of 10,000.

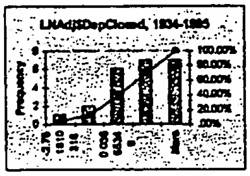
The chart below shows the modern historical log adjusted-dollar values for the four variables: catastrophe losses, statutory underwriting, deposit closings and deposit losses.

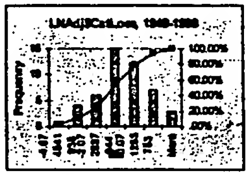
Histograms of Actual $B and LN Values of Variables over Variable History

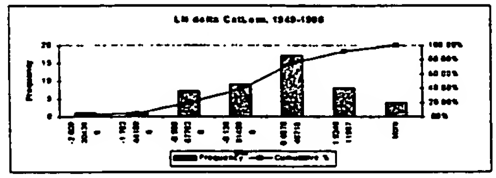




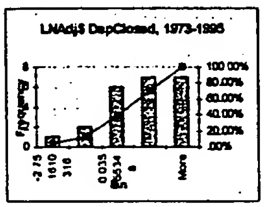


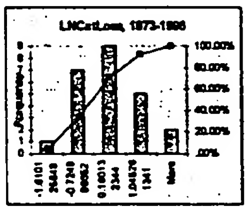


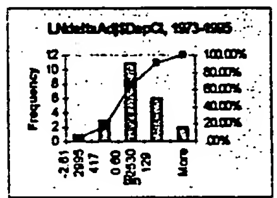


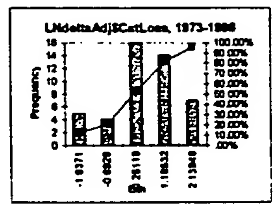


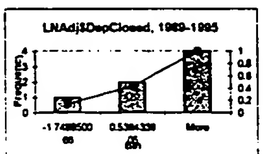
Histograms of LN Values of Variables in Modern Period

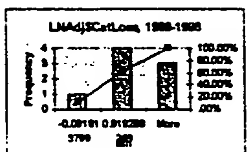


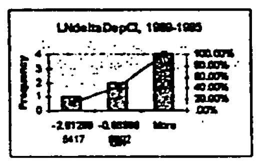


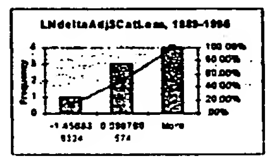


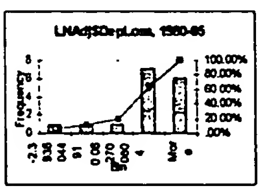


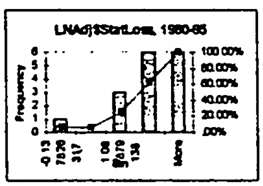


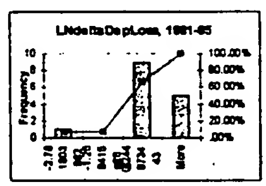


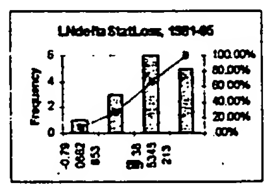




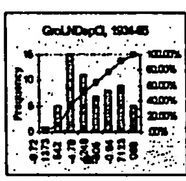


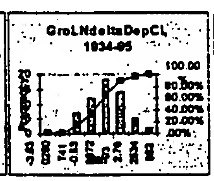


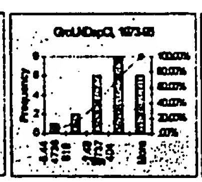


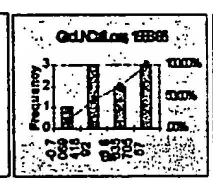


Histograms of Growth Adjusted DepCl and CatLoss, Modern Period



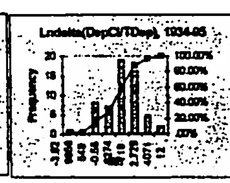




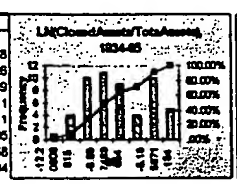


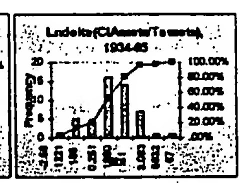
Descriptives and Histograms of (Insured) DepCl/TotalDep and of ClosedAssets/TotalAssets

ClDep/TDep e^([illegible])	
1934 thru 1994	
Mean	1.001959
Standard E	0.000517
Median	1.000221
Mode	1.000295
Standard C	0.00404
Sample Va	1.63E-05
Kurtosis	9.358698
Skewness	3.012877



ClAs/TAs e^([illegible])	
1934 thru 1994	
Mean	1.001868
Standard E	0.000526
Median	1.000109
Mode	1
Standard C	0.004111
Sample Va	1.69E-05
Kurtosis	8.186568
Skewness	2.924594





Histograms of Variables

The statistics on the prospective primary theta variables, deposits cl sed and catastrophe loss, over their historic lives (1934: deposit; 1949: cat), being also the only sample sizes with normal approximation N≥30, show in histogram their lognormal distribution with respect to the actual dollar amounts (Act$B) and to their change (LNdelta) in value. This affords both variables to be modeled via weak convergence, Δt.

With respect to the rigors of continuous time, the log value of theta must exhibit normal bell-shape, indicating lognormal behavior throughout the life, and not just with respect to the year-end terminal value. In this regard, the catastrophe loss variable (LNAdj$) meets this condition visually, while that for the deposit closings do not. The LNAdj$DepClosed distribution function swells in frequency as the value of θ increases.

Thus, for θ(catloss), the full gamut of continuous and discrete time methodologies are reasonably supported. This state of lognormality, with respect to time and with respect to itself, means that in a market with sufficient capital and capacity, the pricing could be expected to be orderly and modeled by Black-Scholes and risk-neutral measures. These early findings suggest that cat loss can underlie robust securities.

For θ(depclosed), the LNAdj$DepClosed, for any of the periods sampled herein, the character is not lognormal with respect to continuity in time. Rather, its histograms resemble the mean residual life functions of Pareto or Weibull (τ<1) distributions. These casualty variable forms emerge in periods of crisis, compare with LNdeltaAdj$CatLoss, 1989-95, which also shows this upward sloping distribution function.

To deal with long-tailed distributions or distributions (in our case) not lognormally distributed, or also to provide frequency and uniformity across the (XS) range of probable values, the data is next scaled, but not transformed. In the "pareto-weibull" periods, 1973 onwards for deposit losses, 1989- for catastrophe loss, the values were adjusted by the growth in the exposure to risk basis, see Growth Adjusted Histograms.

The growth adjusted cat loss was scaled by the 80% rise in insured vulnerable property, 1988-95, with its log d.f. (0.55, 0.66), 1989-95. The deposit closings were scaled by the growth of total insured dollar deposits, 1934-95. The resulting histograms, made primarily for the "crisis", "jump-poisson" periods, are broader across bins, or more normal with respect to their prior distributions. Compare the selected samples.

As the last comment on variables, the descriptives contain overview statistics on two alternative risk variables for deposit insurance and credit default analysis, the insured [Deposits Closed/Total Deposits] and [Closed Assets/Total Assets]. These are portrayed first in exponential form, e^(#/#), those values then shown after conversion, LN[e^(#/#)], to return as broad, frequency rich, probability distribution functions.

Conclusion

This investigation has examined quite a lot of ground and has rendered a number of initial findings. The prospect of a $50 billion insured catastrophe loss is contrasted with about $250 billion surplus capital of the P&C insurers and reinsurers. Such an occurrence could bankrupt many mutual and regional insurers and certainly would devastate the forward underwriting capacity of the entire industry, across every P&C line.

The instruments and general open market means have not yet found sufficient capital to underwrite protection against this loss potential for every insurer. In the absence of substantial new capital for capacity, such a mega-event, or series of continuing painful annual results, could hit the United States economy with a public economic catastrophe, comparable in danger and macro-relational impact to the Great Depression.

This investigation suggests using a portion of the Federal Bank Insurance Fund to capitalize needed capacity. The BIF, a public holding, has the substantial capital needed. Its $27 billion balance greatly exceeds expected forward deposit losses, at least over the near- to mid-term. Current deposit losses, under $1 billion, indicate that most of the BIF balance can r could be applied to cat risk, supplying all of the capital required.

Standardized cat risk reinsurance instruments, capitalized by say $10 billion of the BIF balance. Certificate contracts: divisible, transferable, exchange traded or registered, loss notional or pro rata basis, could provide XS layer or catastrophic event coverage. They could be availed every insurer and reinsurer, with the open and fair trading of the certificates, during their life, to be facilitated by the FDIC and NAIC.

Such instruments could be issued "free", or without initial cost to insurers, if an annual settlement scheme were used, where the holder might pay some charge if the year-end value (loss payment) on the certificate was zero or beneath some sliding threshold. Using methods to discount the year-end receivables, the certificates could be exercised at any time within the term, without necessarily terminating the contract.

All FDIC insured depository institutions (1991: 14,482; 1996: 11,452 - including the Savings Association Insurance Fund - SAIF 1996: balance of $9B; closed assets of $35M) could be released from assessment fees, thereby protecting bank earnings and solvency. Any FDIC insured "problem" bank - 1991: 1,426 (10%); 1996: 117 (1%) - could pay risk assessment fees, booked to their isomorphic solvency benefit.

Hence, on an aggregate BIF+SAIF basis, the FDIC controls public-mandate capital of $36 billion. Aggregate FDIC insured deposit closings for 1996 totaled just $200 million. This is public purpose money that can and should be used to avert catastrophe insurance industry crisis, since it avoids the need for new public underwriting, before or after disaster. These funds are Federal assets that can transcend State license.

Now, the FDIC, as insurer of deposit default loss, pools the nation's entire insured base. As such, it is perfectly positioned to enter into reinsurance, transferring risk to the P&C insurance and reinsurance industry. As organized between the FDIC and the NAIC, with the NAIC acting for the national syndicate, reciprocal treaty reinsurance between the deposit bankers and the P&C insurers can be enacted macro-basis.

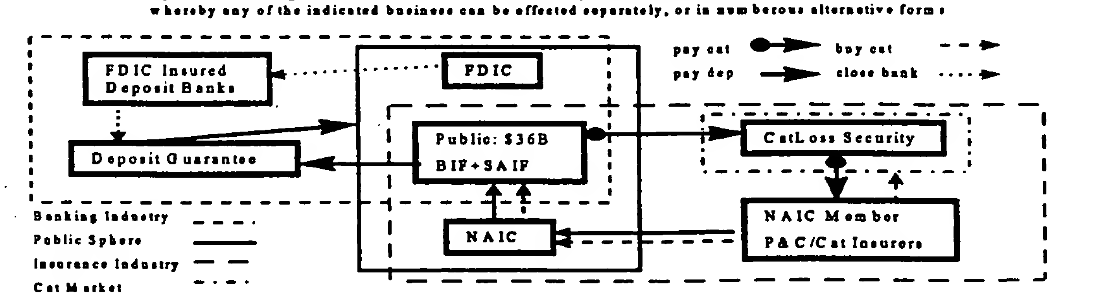
The reciprocal, netting transaction of these disparate insured risks can be modeled as macro-swap. A number of benefits arise out of the insurers reinsuring the FDIC insured banks. The public exposure to "catastrophic" losses from either cat or deposit risk is spread into commercial industry, while netting reduces the public's residual, non-diversified, aggregate exposure. By reinsuring banks, insurers pay less on cat risks.

Also, this reinsurance of deposits by commercial P&C risk-bearers provides the insurance industry with a new line of underwriting that is essential to reduce the portion of cat premiums and liabilities relative within total P&C business. Working directly with the FDIC at the outset will accustom the insurers to the data and perils of deposit loss, whereas the aggregate, regional, State or problem banks can be packaged off.

This inter-industry transfer has a natural case in its favor, since the U.S. public guarantees against loss are on these two forms of society's stored value, real estate and deposit accounts. By enacting the steps suggested herein, the exposure to cat risk is taken on by the BIF+SAIF, and hence, does not directly impact the banks' results; the latter clearly gain in solvency and profitability when assessment rates need not be paid.

The recent discussions in Washington regarding the future of FDIC deposit insurance indicate that the topics and approaches addressed herein are timely and hopefully, contribute to that field of vital inquiry. By introducing commercial insurers to the FDIC guarantee mechanic, this prepares the insurers for the direct insurance of deposits, should policy decide to unwind the FDIC's insurance function from its regulatory role.

Reciprocal Netting Structure of U.S. Insured Deposit and Cat Loss, via FDIC and NAIC
whereby any of the indicated business can be effected separately, or in numberous alternative forms



Notes

About the data used herein, this investigation has relied, wherever possible, on FDIC's Historical Statistics on Banking for insured deposit, closings, asset and industry values, instead of those in the FDIC's Annual Reports. The latter source differs from the former, expressly with reduced totals for the insured deposit population. The PCS annual year-end totals for P&C insured catastrophe loss, 1949-1995, were greatly helpful and were graciously provided by Mr. J. Welsh of AISG.

Time constraints did not allow the math notation to be in standard format. It has aspects of MSExcel formula scripting, i.e. the symbol * stands for multiplication, ^ for exponential. In longer equations, care has been taken with parentheses. All data was entered by author's hand. All charts, tables and computations were created and executed by author. Footnote sources are regrettably only cited in bibliography.

Thanks to

the CIFEr 1998 paper selection committee, the IEEE Neural Networks Council, The College of Insurance, and to the IAFE. My deepest gratitude to the Polytechnic M. Sc. financial engineering program and to my research advisor, Dr. Deane Yang.

Bibliography

A.M. Best. Aggregates & Averages: Property-Casualty. 1996.
American Insurance Association. "New Partnerships for Catastrophe Response". 1995.
Black, F. and Scholes, M. "The Valuation of Option Contracts and A Test of Market Efficiency". *Journal of Finance*. 1972.
Black, F. and Scholes, M. "The Pricing of Options and Corporate Liabilities". *Journal of Political Economy*. #81 May-June. 1973.
Bowers, Gerber, Hickman, Jones and Nesbitt. Actuarial Mathematics. The Society of Actuaries. Schaumburg, IL. 1997.
BZW; Spinner, K. "Supplement: Credit Derivatives". *Derivatives Strategy*. Derivatives Strategy & Tactics LLC. NY. Vol. 2, #6, 6/1997.
Clark, K., Richardson, B. "Catastrophe Reinsurance Underwriting-the Past and Future". Applied Insurance Research. Boston. 1996.
Conning & Company. "Natural Catastrophes: A Growing Cause for Concern". Hartford, CT. 1991.
Doherty, Neil. "Insurance Surplus Shock". NCPI Annual Forum, 1992. Boston.
Dunford, Campbell, ed. The Handbook of International Trade Finance. Woodhead-Faulkner, Ltd. New York. 1991.
D'Zmura, David Andrew. "Analysis of Regulatory Impact on U.S. Commercial Banking". Unpublished. 1996.
D'Zmura, David Andrew. "Investigation of U.S. Depository Banking and P&C Catastrophe Insurance Industries". M.Sc.FE. 1997.
Elliot, Margaret. "Convergence or Collision?". *Derivatives Strategy*. Derivatives Strategy & Tactics LLC. NY. Vol. 3, #1.12/97.
FDIC. Historical Statistics on Banking. Annual Editions. 1994, 1995. Washington, D.C.
FDIC. Annual Reports, 1994, 1995 and 1996. Washington, D.C.
FDIC. "Quarterly Banking Profile". Fourth Quarter 1996. Washington, D.C.
FDIC. "Executive Management Report: Financial Results, 1996". Division of Finance. Washington, D.C.
Gart, Alan. Regulation, Deregulation, Reregulation. Wiley & Sons. New York. 1994.
Gastineau, Gary. Dictionary of Financial Risk Management. Probus Publishing Co. Chicago. 1992.
Gottheimer, George. Re-Concepts: Principles of Reinsurance. MBA Executive Edition. Kernan Press. Berkeley Heights, NJ. 1997.
Gottlieb, Paul. "Regulatory Framework for Derivative Instruments under U. S. Law". *Derivatives Use, Trading, Regulation*. Vol. 1, # 4.
Herzog, Thomas. Introduction to Credibility Theory. ACTEX Publications, Inc. Winsted, CT. 2nd Ed. 1996.
Hogg, R. and Klugman, S. Loss Distributions. John Wiley & Sons. New York. 1984.
Howard, C. D. "Lectures on Continuous Time Mathematics in Finance". Polytechnic University, N.Y. 1997.
Hull, John. Options, Futures and Other Derivative Securities. Prentice Hall. New Jersey. Second Edit. 1993.
Insurance Information Institute. Fact Book. New York. Years 1995-1997.
Insurance Services Office, Inc. "The Impact of Catastrophes on Property Insurance". New York. 1994.
Insurance Services Office, Inc. "Managing Catastrophe Risk". New York. 1996.
James and James. Mathematics Dictionary. D. Van Nostrand. New York. 1959.
Jarrow and Turnbull. Derivative Securities. South-Western College Publishing. Cincinnati, Ohio. 1996.
Jacob, Klaus. "Risk from Earthquakes in the Eastern United States". NCPI Annual Forum, 1990. Boston.
Kleffner, A.E. "Catastrophic Risks and the Functioning of Insurance Markets". UMI Dissertation. 1993.
Kunreuther, Howard. "The Micro and Macro Impacts of the Catastrophic Earthquake" NCPI Annual Forum, 1991. Baltimore.
Marshall, John, et al. "Hedging Business Cycle Risks with Macro Swaps and Options". *Journal of Applied Corporate Finance*.
Marshall, J. "Macroeconomic Derivatives: Tools for Hedging Business-Cycle Risk". *Financial Derivatives and Risk Management*. 1996.
Merton, Robert. Continuous Time Finance. Blackwell. Cambridge, MA. 1992.
Montross, Franklin. "The Catastrophe Loss Potential: How Can the Industry Respond?". NCPI Annual Forum, 1992. Boston.
Property Claims Services, Inc. Data: P&C Insured Catastrophe Losses. American Insurance Services Group. New Jersey. 1997.
Rothstein, Nancy, ed. The Handbook of Financial Futures. McGraw-Hill Book Company. New York. 1984.
Shah, Haresh. "Scientific Profiles of the "Big One"". Insurance Institute Congress. Georgia. 1995.
Spinner, Karen. "Managing Bank Loan Risk". *Derivatives Strategy*. Derivatives Strategy & Tactics LLC. New York. Vol. 3, #1, 1/1998.
Thackray, John. "Insurance Derivatives Catch Fire". *Derivatives Strategy*. Derivatives Strategy & Tactics LLC. NY. Vol. 2,#1.Dec/Jan97.
Tuckman, Bruce. Fixed Income Securities. John Wiley & Sons. New York. 1995.
Wunnicke, Brooke and Diane Wunnicke. Standby Letters of Credit. John Wiley & Sons. New York. 1989.

"Historical Statistics on Banking": FDIC

$ in billions

	Bank Closings			Insured Commercial Banks and Trusts								
Year End	# Banks	Deposits	Assets	Total Assets	Total Deposits	Interest-Bearing (IB) Deposits	IB/TotalDe	Total # Banks (000's)	Total Assets	I-B Assets	I-B Liabilities	Total Liabilities
12/31/94	12	1.238	1.392	4010.6	2874.4	2302	0.801	10.5	4010.6	3568.6	3023.8	
12/31/93	41	3.132	3.539	3706.2	2754.3	2182	0.792	11	3706.2	3290.8	2717.5	
12/31/92	122	40.94	44.232	3505.7	2698.7	2157	0.799	11.5	3505.7	3087.9	2598.3	
12/31/91	127	53.832	63.338	3430.6	2687.6	2207	0.821	11.9	3430.6	3011.1	2611.9	
12/31/90	169	14.489	15.365	3389.5	2650.1	2161	0.815	12.3	3389.5	2956.3	2570.7	
12/31/89	207	22.28	29.431	3299.4	2548.5	2065	0.810	12.7	3299.4	2887.9	2503.9	
12/31/88	221	36.432	52.62	3130.8	2431.7	1952	0.803	13.1	3130.8	2734.5	2350.5	
12/31/87	203	8.4	9.216	2999.9	2335.4	1857	0.795	13.7	2999.9	2627	2237.2	
12/31/86	145	7.057	6.813	2940.7	2283.5	1751	0.767	14.2	2940.7	2548.6	2127	
12/31/85	120	8.059	8.735	2730.7	2118.1	1646	0.777	14.4	2730.7	2362.7	1981.9	
12/31/84	80	20.334	36.909	2508.9	1962.9	1531	0.780	14.5	2508.9	2157	1803	
12/31/83	48	5.442	7.026	2342.1	1842.5	1451	0.788	14.5	2342.1	2005.7	1696.4	
12/31/82	42	9.904	11.632	2193.3	1705.7	1335	0.783	14.4	2193.3	1880.2	1580.4	
12/31/81	10	3.826	4.99	2028.9	1588.7	1205	0.758	14.4	2028.9	1725.5	1422.9	
12/31/80	11	5.516	8.189	1855.7	1481.1	1049	0.708	14.4	1855.7	1551.9	1233.9	
12/31/79	10	0.111	0.133	1691.8	1362.8	932	0.684	14.4	1691.8	1405.9	1092.6	
12/31/78	7	0.854	0.994	1507.9	1233.4	834	0.676	14.4	1507.9	1257.3	968.8	
12/31/77	6	0.205	0.233	1339.4	929.2	550	0.592	14.4	1339.4	1131.2	841.2	
12/31/76	17	1.235	1.039	1182.4	830.9	497	0.598	14.4	1182.4	1013.9	743.9	
12/31/75	13	0.34	0.42	1086.7	780.7	459	0.588	14.4	1086.7	920.7	660.2	
12/31/74	4	1.576	3.823	1037.2	746.4	432	0.579	14.2	1037.2	869	621.3	
12/31/73	6	0.971	1.31	820.4	681.7	373	0.547	14	820.4	673.6	434.9	
12/31/72	2	0.02	1.322	730.9	616.9	320	0.519	13.7	730.9	595	383.4	
12/31/71	7	0.141	0.197	633.6	539.2	277	0.514	13.6	633.6	513.7	306.2	
12/31/70	7	0.052	0.062	570.2	482.5	236	0.489	13.5	570.2	456.9	257.3	
12/31/69	9	0.04	0.044	524.6	436.9	197	0.451	13.5	524.6	416.3	217.5	
12/31/68	3	0.023	0.025	500.2	434.6	206	0.474	13.5	500.2	401.7	216.7	
12/31/67	4	0.011	0.012	450.6	395.8	185	0.467	13.5	450.6	360.7	192.9	
12/31/66	7	0.104	0.121	402.9	352.8	161	0.456	13.5	402.9	323.2	167.6	
12/31/65	5	0.044	0.059	375.4	331.5	148	0.446	13.5	375.4	305.5	153.7	
12/31/64	7	0.023	0.026	345.1	306.2	127	0.415	13.5	345.1	276.9	130.9	
12/31/63	2	0.023	0.026	311.8	274.6	111.7	0.407		311.8	254.1	115.4	
12/31/62	1	0.003	0	295.9	261.4	98.2	0.376		295.9	235.4	101.8	
12/31/61	5	0.009	0.01	277.3	247.9	82.8	0.334		277.3	214.8	83.3	
12/31/60	1	0.007	0.008	256.3	228.9	73.3	0.320		256.3	198.7	73.5	
12/31/59	3	0.003	0.003	243.4	219	67.5	0.308		243.4	189.4	68.1	
12/31/58				237.5	215.2	66.7	0.305		237.5	184.4	65.8	
12/31/57	2	0.011	0.011	221.5	200.5	57.6	0.287		221.5	169.2	57.8	
12/31/56	2	0.011	0.013	216.1	196.5	52.1	0.265		216.1	164.2	52.2	
12/31/55	5	0.012	0.012	209.1	190.9	49.9	0.281		209.1	159.7	50.1	
12/31/54	2	0.001	0.001	200.6	183.3	48.5	0.265		200.6	154.6	48.6	
12/31/53	4	0.044	0	191.1	175.1	44.8	0.256		191.1	144.2	44.9	
12/31/52	3	0.003	0.002	186.7	171.4	41.4	0.242		186.7	140.2	41.6	
12/31/51	2	0.003	0.003	177.5	163.2	38.3	0.235		177.5	131.1	38.3	
12/31/50	4	0.006	0.004	166.7	153.5	36.5	0.238		166.7	125.2	36.6	
12/31/49	5	0.006	0.005	155.3	143.2	36	0.251		155.3	116.4	36	
12/31/48	3	0.01	0.01	152.1	140.7	35.5	0.252		152.1	112.4	35.6	
12/31/47	5	0.007	0.007	152.7	141.9	34.9	0.246		152.7	114.4	35	
12/31/46	1	0.001	0.001	147.3	137	33.6	0.245		147.3	112.3	33.7	
12/31/45	1	0.006	0.006	157.5	147.8	29.9	0.202		157.5	121.9	30.2	
12/31/44	2	0.002	0.002	134.6	125.7	23.9	0.190		134.6	103.5	24.1	
12/31/43	5	0.012	0.014	112.2	104.1	19.2	0.184		112.2	83.6	19.2	
12/31/42	20	0.017	0	95.5	87.8	16.3	0.186		95.5	66.4	16.3	
12/31/41	15	0.03	0	76.8	69.4	15.9	0.229		76.8	49.5	15.9	
12/31/40	43	0.144	0	70.7	63.5	15.7	0.247		70.7	42.7	15.8	
12/31/39	60	0.161	0	63.1	56.1	15.2	0.271		63.1	39.5	15.3	
12/31/38	74	0.062	0.014	56.8	49.8	14.8	0.297		56.8	37.7	14.9	
12/31/37	77	0.033	0.019	54.2	47.2	14.8	0.314		54.2	37.4	14.9	
12/31/36	69	0.028	0.012	56.2	49.3	14.1	0.286		56.2	38.5	14.2	
12/31/35	26	0.013	0.012	50.9	44.1	13.4	0.304		50.9	35.1	13.4	
12/31/34	9	0.002	0.003	46.4	39	12.7	0.326		46.4	33.1	12.8	

	Bank Closings versus Total Industry				Ratio of Deposits versus Assets/Liabilities			
	Closed Deposits/ Total Deposits		Closed Assets/ Total Assets		Total Deposits vs. Assets	Total Deposits vs. Liabilities	I-B Deposits vs I-B Asset	I-B Deposits vs I-B Liabls
Year End	##	e^(##)	##	e^(##)	TD/TA	TD/TL	IBD/IBA	IBD/IBL
12/31/94	0.00043	1.00043	0.00035	1.000347	0.717		0.645	0.761
12/31/93	0.00114	1.001138	0.00095	1.000955	0.743		0.663	0.803
12/31/92	0.01517	1.015286	0.01262	1.012697	0.770		0.699	0.830
12/31/91	0.02003	1.020232	0.01846	1.018634	0.783		0.733	0.845
12/31/90	0.00547	1.005482	0.00453	1.004543	0.782		0.731	0.841
12/31/89	0.00874	1.008781	0.00892	1.00896	0.772		0.715	0.825
12/31/88	0.01498	1.015095	0.01681	1.016949	0.777		0.714	0.830
12/31/87	0.00360	1.003603	0.00307	1.003077	0.778		0.707	0.830
12/31/86	0.00309	1.003095	0.00232	1.002319	0.777		0.687	0.823
12/31/85	0.00380	1.003812	0.00320	1.003204	0.776		0.697	0.831
12/31/84	0.01036	1.010413	0.01471	1.01482	0.782		0.710	0.849
12/31/83	0.00295	1.002958	0.00300	1.003004	0.787		0.723	0.855
12/31/82	0.00581	1.005823	0.00530	1.005318	0.778		0.710	0.845
12/31/81	0.00241	1.002411	0.00246	1.002462	0.783		0.698	0.847
12/31/80	0.00372	1.003731	0.00441	1.004423	0.798		0.676	0.850
12/31/79	0.00008	1.000081	0.00008	1.000079	0.806		0.663	0.853
12/31/78	0.00069	1.000693	0.00066	1.000659	0.818		0.663	0.863
12/31/77	0.00022	1.000221	0.00017	1.000174	0.694		0.486	0.654
12/31/76	0.00149	1.001487	0.00088	1.000879	0.703		0.490	0.668
12/31/75	0.00044	1.000438	0.00039	1.000387	0.718		0.499	0.695
12/31/74	0.00211	1.002114	0.00369	1.003693	0.720		0.497	0.695
12/31/73	0.00142	1.001425	0.00160	1.001598	0.831		0.554	0.858
12/31/72	0.00003	1.000032	0.00181	1.00181	0.844		0.538	0.881
12/31/71	0.00026	1.000262	0.00031	1.000311	0.851		0.539	0.905
12/31/70	0.00011	1.000108	0.00011	1.000109	0.846		0.517	0.917
12/31/69	0.00009	1.000092	0.00008	1.000084	0.833		0.473	0.906
12/31/68	0.00005	1.000053	0.00005	1.00005	0.869		0.513	0.951
12/31/67	0.00003	1.000028	0.00003	1.000027	0.878		0.513	0.959
12/31/66	0.00029	1.000295	0.00030	1.0003	0.876		0.498	0.961
12/31/65	0.00013	1.000133	0.00016	1.000157	0.883		0.484	0.963
12/31/64	0.00008	1.000075	0.00008	1.000075	0.887		0.459	0.970
12/31/63	0.00008	1.000084	0.00008	1.000083	0.881		0.440	0.968
12/31/62	0.00001	1.000011	0.00000	1	0.883		0.417	0.965
12/31/61	0.00004	1.000036	0.00004	1.000036	0.894		0.385	0.994
12/31/60	0.00003	1.000031	0.00003	1.000031	0.893		0.369	0.997
12/31/59	0.00001	1.000014	0.00001	1.000012	0.900		0.356	0.991
12/31/58	0.00000	1	0.00000	1	0.906		0.356	0.998
12/31/57	0.00005	1.000055	0.00005	1.00005	0.905		0.340	0.997
12/31/56	0.00006	1.000056	0.00006	1.00006	0.909		0.317	0.998
12/31/55	0.00006	1.000063	0.00006	1.000057	0.913		0.312	0.998
12/31/54	0.00001	1.000005	0.00000	1.000005	0.914		0.314	0.998
12/31/53	0.00025	1.000251	0.00000	1	0.916		0.311	0.998
12/31/52	0.00002	1.000018	0.00001	1.000011	0.918		0.295	0.995
12/31/51	0.00002	1.000018	0.00002	1.000017	0.919		0.292	1.000
12/31/50	0.00004	1.000039	0.00002	1.000024	0.921		0.292	0.997
12/31/49	0.00004	1.000042	0.00003	1.000032	0.922		0.304	1.000
12/31/48	0.00007	1.000071	0.00007	1.000066	0.925		0.316	0.997
12/31/47	0.00005	1.000049	0.00005	1.000046	0.929		0.305	0.997
12/31/46	0.00001	1.000007	0.00001	1.000007	0.930		0.299	0.997
12/31/45	0.00004	1.000041	0.00004	1.000038	0.938		0.245	0.990
12/31/44	0.00002	1.000016	0.00001	1.000015	0.934		0.231	0.992
12/31/43	0.00012	1.000115	0.00012	1.000125	0.928		0.230	1.000
12/31/42	0.00019	1.000194	0.00000	1	0.919		0.245	1.000
12/31/41	0.00043	1.000432	0.00000	1	0.904		0.321	1.000
12/31/40	0.00227	1.00227	0.00000	1	0.898		0.368	0.994
12/31/39	0.00287	1.002874	0.00000	1	0.889		0.385	0.993
12/31/38	0.00124	1.001246	0.00025	1.000247	0.877		0.393	0.993
12/31/37	0.00070	1.000699	0.00035	1.000351	0.871		0.398	0.993
12/31/36	0.00057	1.000568	0.00021	1.000214	0.877		0.366	0.993
12/31/35	0.00029	1.000295	0.00024	1.000238	0.868		0.382	1.000
12/31/34	0.00005	1.000051	0.00006	1.000065	0.841		0.384	0.992

Method and Process f Small Sample Technology

The method and process of the small sample technology create numerical data and statistical tools which benchmark the character of distribution functions in small sample environments and afford means for the evaluation of the nature of financial data with respect to establishing its underlying distribution character.

The method and process of the small sample technology is useful as it simulates and demonstrates how a uniform random variable with a known distribution function occurs in small sample environments, i.e. in the absence of the large data sets by which the Law of Large Numbers is applicable in shaping frequency.

The method and process of the small sample technology can be used for any set of discrete data, and is useful for data sets which are a time-series or are serially correlated. Inventor's examples of small sample financial data are annual U.S. insured depository default and insured catastrophic property casualty losses.

The method and process of the small sample technology is especially designed for evaluation of data which may be a standard normal variable. Financial pricing formulas and theories imply, infer or require that the financial data is a standard or log normal variable. It can be applied to other disciplines' data sets.

The method and process of the small sample (typically $N \leq 30$, can be greater) technology comprises:

A) random generation of sequences of independent uniform random variables as separate data arrays, over the values of zero to one, each sequence separately seeded, each using a different seed clock rate;

B) taking the data arrays in pairs and performing a Box-Muller transformation on the two data arrays, using the formulas, one for each array, to generate numeric output of standard normal random variables:
Box-Muller: Standard Normal Random Variable V1 = SQRT(-2×LN(U(Ia))) × COS(2×PI×(U(Ib)))
Box-Muller: Standard Normal Random Variable V2 = SQRT(-2×LN(U(Ia))) × SIN(2×PI×(U(Ib))),
where the Ia and Ib are from each of the two distinct, paired, data arrays;

C) utlizing the numeric output of a transformed data array, by taking small samples of its data to generate descriptive statistics of the small samples, which can then be used to indicate the probable and occurring statistical characteristics of standard normal variables subjected to small sample environments, which further comprises:
- a) generating, minimally, the mean, standard deviation and sample variance of the samples;
- b) generating a histogram for the sample, of its frequency with respect to range of bin values;
- c) generating descriptive statistics for various sized small samples of the various data arrays;

D) utilizing the statistical descriptive characteristics of the small samples generated to evaluate and serve as benchmark to the nature of the distribution underlying financial or other data in small sample sets.

The method and process of the small sample technology further comprises alternate lognormal technology:
A) taking the natural log of the transformed data arrays, ie., this done after the step B) above,
B) utilizing log numeric output to generate descriptive statistics and to evaluate distributions of data.

The method and process of the small sample technology further comprises alternate transform technology:
A) taking the data arrays in singleton and using the formula for each array, generating output:
Random Variable = SQRT(-2×LN(U(Ia))) × COS(2×PI×(U(Ib))),
where the Ia and Ib are from the same sequence data array;
B) utilizing a series formalism to relate the Ia and the Ib, such as Ui and Ui+1 respectively.
This latter type of generated random variable tests in large numbers as standard normal with respect to mean (of zero), standard deviation (of one) and variance (of one), irrespective of the relational formalism.

10 S quences of Ind pendent Uniform Random Variabl s on (0,1)
Each Sequence with Different S d Clock Rat

0.382	0.655507	0.894681	0.670064	0.267006	0.436171	0.672079	0.682669	0.817438
0.100681	0.01825	0.803186	0.609638	0.456893	0.46028	0.326518	0.770135	0.326426
0.596484	0.54442	0.608997	0.666982	0.136814	0.106754	0.822169	0.469069	0.421735
0.899106	0.208106	0.477767	0.745323	0.523515	0.135258	0.938231	0.353069	0.620075
0.88461	0.734306	0.8717	0.62392	0.058046	0.272378	0.341227	0.664327	0.019074
0.958464	0.372997	0.612537	0.552904	0.94821	0.313913	0.698569	0.623615	0.999817
0.014496	0.998077	0.352062	0.854305	0.563677	0.968749	0.967925	0.218543	0.545366
0.407422	0.420728	0.557237	0.837062	0.05829	0.751366	0.514237	0.059633	0.882534
0.863247	0.994873	0.240364	0.2725	0.45497	0.40611	0.176885	0.025178	0.138371
0.138585	0.038575	0.085757	0.181555	0.411969	0.20304	0.689779	0.061922	0.501938
0.245033	0.231605	0.99353	0.207984	0.234107	0.721549	0.931394	0.452071	0.718406
0.045473	0.312296	0.053133	0.168188	0.785485	0.359416	0.517777	0.379711	0.54564
0.03238	0.694113	0.681448	0.768456	0.286569	0.601428	0.369366	0.005127	0.940367
0.164129	0.367962	0.407544	0.227638	0.192389	0.701071	0.848903	0.295785	0.708365
0.219611	0.315806	0.798059	0.462203	0.123966	0.547655	0.207404	0.246956	0.303262
0.01709	0.782281	0.214637	0.245766	0.252541	0.203711	0.540849	0.452803	0.055757
0.285043	0.298135	0.817225	0.962249	0.984985	0.250984	0.41258	0.570788	0.984924
0.343089	0.969085	0.102298	0.524979	0.268868	0.753685	0.383984	0.769463	0.742454
0.553636	0.907682	0.519089	0.459639	0.967345	0.739494	0.601978	0.737815	0.019898
0.357372	0.916715	0.301584	0.009033	0.740349	0.518754	0.375134	0.973327	0.48207
0.371838	0.877468	0.8081	0.134159	0.759117	0.816034	0.803034	0.844569	0.877529
0.355602	0.144566	0.368755	0.438093	0.36079	0.983489	0.690939	0.999145	0.848933
0.910306	0.056795	0.019898	0.989959	0.067537	0.910184	0.771722	0.225929	0.451277
0.466018	0.194372	0.84582	0.939055	0.45439	0.522324	0.018006	0.98703	0.625111
0.42616	0.613422	0.193762	0.79104	0.145207	0.971557	0.146702	0.339885	0.068575
0.303903	0.997406	0.682211	0.09769	0.901151	0.879635	0.657247	0.99176	0.620289
0.975707	0.013489	0.092013	0.208472	0.501114	0.549638	0.437422	0.548631	0.488235
0.806665	0.687094	0.32075	0.427717	0.538804	0.371929	0.657064	0.908322	0.306894
0.991241	0.322245	0.509445	0.683126	0.090213	0.862728	0.330729	0.534776	0.108066
0.256264	0.483566	0.070315	0.973998	0.410718	0.653493	0.772454	0.649678	0.858394
0.951689	0.195227	0.252358	0.041047	0.12009	0.583026	0.177801	0.489456	0.870205
0.053438	0.785485	0.875484	0.18833	0.693625	0.114994	0.914212	0.41438	0.374126
0.705039	0.403027	0.708792	0.698569	0.225074	0.37907	0.915494	0.814325	0.761254
0.816523	0.19367	0.969451	0.819178	0.303964	0.041353	0.576586	0.0683	0.851924
0.972503	0.967589	0.799829	0.481521	0.259957	0.086978	0.209204	0.510727	0.477798
0.466323	0.481216	0.289407	0.987945	0.676809	0.164617	0.131077	0.372478	0.445173
0.300211	0.115421	0.924833	0.790551	0.483352	0.85226	0.403821	0.313425	0.892148
0.750206	0.143864	0.066897	0.510361	0.657521	0.835994	0.477676	0.132328	0.350993
0.351482	0.395703	0.913297	0.187963	0.558275	0.820704	0.058138	0.58977	0.533616
0.775658	0.737419	0.803919	0.483413	0.472396	0.331797	0.976684	0.456893	0.932798
0.074343	0.594226	0.599994	0.594287	0.722037	0.083926	0.543413	0.691488	0.520798
0.198431	0.358165	0.03592	0.824213	0.973174	0.427503	0.469497	0.317179	0.020631
0.064058	0.623096	0.071444	0.284249	0.791559	0.727195	0.53267	0.439436	0.367504
0.358348	0.977783	0.987182	0.442457	0.405835	0.94583	0.158147	0.611316	0.373272
0.487045	0.506027	0.516434	0.889431	0.500504	0.110294	0.318522	0.369823	0.960662
0.511216	0.522752	0.569597	0.674123	0.465926	0.135838	0.781304	0.101627	0.653584
0.373455	0.032991	0.576525	0.772698	0.13538	0.062258	0.89877	0.89526	0.902554

0.9859	0.226661	0.288583	0.130161	0.987854	0.093539	0.516861	0.013398	0.593677
0.040712	0.755028	0.758141	0.564562	0.673788	0.608661	0.787652	0.2631	0.442
0.23072	0.493881	0.619617	0.842067	0.909085	0.614948	0.990112	0.898343	0.924345
0.004975	0.849971	0.462447	0.464248	0.755058	0.305918	0.667898	0.749321	0.51793
0.926145	0.930265	0.438734	0.143345	0.768029	0.509232	0.290139	0.697501	0.747917
0.100314	0.143681	0.135746	0.239967	0.956755	0.353313	0.422864	0.536515	0.128574
0.256691	0.940825	0.24662	0.735313	0.566637	0.110782	0.627369	0.90762	0.965575
0.775689	0.336467	0.569353	0.832759	0.132206	0.513657	0.902585	0.812983	0.445021
0.679647	0.991028	0.35139	0.473098	0.979614	0.14594	0.820399	0.491684	0.753563
0.809107	0.39494	0.105777	0.204627	0.115085	0.626301	0.107639	0.419172	0.932249
0.724326	0.904233	0.987976	0.200201	0.24546	0.85406	0.810877	0.801996	0.547838
0.085055	0.24189	0.065096	0.699149	0.034303	0.263436	0.445296	0.489273	0.35313
0.132267	0.04593	0.683676	0.07709	0.256264	0.959807	0.492721	0.425642	0.160009
0.756157	0.441816	0.714927	0.824244	0.939695	0.989349	0.448866	0.605701	0.033357
0.626514	0.552385	0.149174	0.923582	0.997467	0.368877	0.933287	0.646474	0.14423
0.17365	0.659597	0.195349	0.15183	0.532792	0.854488	0.977844	0.636311	0.280557
0.404798	0.520646	0.710685	0.743156	0.202612	0.938444	0.182318	0.265419	0.979797
0.552324	0.390851	0.601276	0.075747	0.660085	0.323679	0.559679	0.276833	0.512223
0.711509	0.915647	0.926023	0.453291	0.503433	0.97882	0.938749	0.961821	0.660939
0.555162	0.948363	0.543596	0.610096	0.976257	0.914243	0.316599	0.002838	0.567949
0.181158	0.4897	0.670827	0.524705	0.099429	0.904904	0.99292	0.331553	0.513627
0.970275	0.065889	0.583239	0.365612	0.9541	0.543565	0.96942	0.270486	0.998077
0.686941	0.880581	0.762139	0.369518	0.669973	0.291299	0.218482	0.336375	0.114933
0.528794	0.874569	0.581896	0.435713	0.325144	0.07007	0.8511	0.01764	0.949675
0.796686	0.874905	0.238136	0.530198	0.524796	0.128056	0.567309	0.256325	0.701529
0.805658	0.537736	0.867672	0.287057	0.209021	0.889737	0.81637	0.434339	0.741966
0.262215	0.225349	0.84994	0.637104	0.470321	0.942015	0.656758	0.49321	0.040712
0.177953	0.631581	0.794183	9.16E-05	0.75399	0.841426	0.530473	0.213263	0.459731
0.866756	0.01294	0.277841	0.346904	0.172582	0.271004	0.751701	0.837062	0.029756
0.114841	0.54677	0.408399	0.473312	0.191321	0.209418	0.980468	0.691427	0.019745
0.059511	0.977355	0.179846	0.671285	0.560747	0.379955	0.858577	0.218329	0.906125
0.761559	0.891812	0.152867	0.612781	0.927183	0.408734	0.305795	0.066103	0.30839
0.738395	0.123295	0.541093	0.880856	0.649556	0.708518	0.894589	0.641072	0.526383
0.986297	0.224311	0.446486	0.202734	0.780572	0.128758	0.311136	0.84991	0.27134
0.925596	0.873989	0.045045	0.147313	0.703543	0.97528	0.090274	0.056703	0.319529
0.903867	0.548723	0.230323	0.217109	0.855281	0.416272	0.433302	0.533647	0.385601
0.544969	0.707633	0.455794	0.894253	0.531968	0.744957	0.561693	0.519211	0.875393
0.500778	0.018921	0.763146	0.912015	0.924039	0.524155	0.607837	0.687948	0.882656
0.674978	0.208777	0.407025	0.085665	0.527848	0.230079	0.964141	0.181921	0.309488
0.489822	0.304361	0.726341	0.26014	0.737327	0.463485	0.545396	0.07007	0.547899
0.145787	0.412275	0.669179	0.020966	0.463546	0.763848	0.948363	0.68337	0.181677
0.037965	0.234138	0.676901	0.274026	0.468581	0.198645	0.507889	0.345622	0.016053
0.796258	0.843135	0.029237	0.243599	0.659688	0.374462	0.398541	0.34608	0.457381
0.67156	0.758812	0.692831	0.642415	0.695791	0.442885	0.895199	0.40202	0.344401
0.731681	0.38493	0.498581	0.542375	0.575304	0.840266	0.769982	0.505112	0.41731
0.584521	0.083743	0.316355	0.996277	0.025422	0.169652	0.939573	0.554552	0.396527
0.152226	0.665273	0.784326	0.344829	0.752251	0.128056	0.88464	0.315989	0.165288
0.892178	0.15772	0.237159	0.154149	0.810419	0.379284	0.681661	0.844935	0.338908
0.377819	0.656423	0.946745	0.703482	0.009278	0.847468	0.912839	0.691031	0.080905
0.200476	0.671072	0.199316	0.996216	0.5244	0.474258	0.295358	0.736473	0.458327

Numeric Output f B x-Muller Transf rmati n on Unif rm R.V. Sample Sequenc s

Ten Uniform Sampling Sequences, each separately seeded; made standard normal by Box-Muller Method:

Box-Muller: Standard Normal Random Variable V1 = SQRT(-2*LN(U(Ia))) * COS(2*PI*(U(Ib)))

Box-Muller: Standard Normal Random Variable V2 = SQRT(-2*LN(U(Ia))) * SIN(2*PI*(U(Ib)))

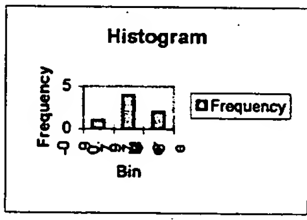
Pair A:		**Pair B:**		**Pair C:**		**Pair D:**		**Pair E:**	
V1	V2	V1	V2	V1	V2	V1	V2	V1	V2
-0.77614	0.62068	-0.22712	-0.41352	-1.49615	0.634424	-0.366	-0.8129	0.560542	-0.29825
2.128731	1.673026	-0.51109	-0.42086	-1.21287	0.30914	0.18877	-1.48422	-0.48062	-1.41708
-0.97723	-0.62832	-0.49626	-0.86349	1.562444	1.239769	-0.61401	0.120856	-1.04354	0.798606
0.120006	-1.04951	-0.03572	-1.2149	0.750996	0.854626	-0.21543	0.284799	-0.97657	0.046197
-0.04875	-0.52118	-0.37306	-0.36803	-0.33437	2.362462	-0.75181	-1.25905	-1.35373	2.467037
-0.20336	-0.36238	-0.9359	-0.32308	-0.12747	0.300181	-0.60412	-0.5937	0.010386	-0.01607
2.909723	0.005643	0.880622	-1.1456	1.050193	-0.20891	0.050141	0.250374	-1.08771	-0.1717
-1.17726	0.722988	0.562509	-0.92364	0.020449	-2.38416	1.073304	0.422092	-0.47982	0.1403
0.542037	-0.07679	-0.23791	1.671706	-1.04289	0.698167	1.838077	0.293227	-0.85046	1.797873
1.929991	1.951441	0.92407	2.014589	0.387277	1.274217	0.797428	0.326918	-1.14892	0.241972
0.193411	1.70956	0.029731	0.109991	-0.30301	-1.67694	-0.36005	0.111832	0.386931	0.71535
-0.94849	0.429991	1.191293	2.109679	-0.44099	0.537062	-0.83496	0.786935	1.100508	0.021739
-0.90097	0.17266	0.101332	-0.86994	-1.27067	-0.94072	1.410628	0.045458	0.337296	0.095927
-1.28354	1.213177	0.187642	1.326651	-0.54943	-1.73049	-0.1624	0.54886	-0.26612	-0.78662
-0.69959	1.490714	-0.65282	0.15802	-1.95249	-0.60273	0.033927	1.773422	-0.08476	-1.54244
0.574655	0.075106	0.046672	1.753693	0.475742	1.589351	-1.06031	0.323988	2.027779	1.289033
-0.47191	1.51821	0.617569	-0.1493	-0.00108	0.173945	-1.2012	-0.57252	-0.12157	-0.12491
1.435203	0.208953	-2.10911	-0.33376	0.037519	-1.62039	0.168773	-1.37325	-0.20174	-0.74491
0.909556	-0.14554	-1.10852	0.287302	-0.017	-0.25712	-0.07706	-1.00456	-2.59152	-1.05749
1.242578	0.325676	1.545865	0.087836	-0.77004	-0.09116	1.380717	-0.2336	-0.38582	-1.14476
1.009935	0.368658	0.434281	0.487381	0.299273	-0.67944	0.370814	-0.54883	-0.04017	-0.50958
0.884465	1.549458	-1.30701	0.535688	1.42023	-0.14787	0.859876	-0.00462	0.349495	0.453213
0.406218	-1.27949	2.793407	-0.17648	1.961681	-1.24175	0.108469	0.711686	0.182174	1.248263
0.423176	0.383533	0.536788	-0.21623	-1.24369	-0.1756	2.825042	-0.23075	0.846784	-0.47182
-0.98823	0.442399	0.461998	-1.7518	1.933192	-0.34922	-1.04862	1.65502	2.283682	-0.38006
1.543197	0.067981	0.714911	0.503722	0.331875	-0.31309	0.914955	-0.04742	0.812076	0.543759
0.220982	-0.4462	0.563531	2.110468	-1.11881	-0.36071	-1.2264	-0.38685	-0.69911	0.972192
-0.2524	-0.81202	-1.35516	0.661603	-0.77107	0.801414	0.768594	-0.49921	0.747535	-1.34302
-0.05816	-0.08279	-0.47377	-1.06038	1.426902	-1.66587	-1.45222	-0.32246	1.321565	-1.64423
-1.64139	1.204527	2.273561	-0.37479	-0.76027	-1.09622	-0.42355	-0.58049	0.107372	0.542084
0.106178	-0.54029	1.604577	0.42326	-1.78504	-1.02599	-1.85447	0.123045	-0.0366	-0.52604
0.535192	0.228965	0.194867	0.477475	0.641641	0.565637	-0.36371	0.217017	1.225079	0.68228
-0.68562	-1.29471	-0.26348	-0.78674	-1.25203	1.189573	0.16525	-0.38636	-0.17704	-0.7171
0.220676	-1.65598	0.104895	-0.22594	1.491472	0.396485	0.954252	0.436651	-0.04276	0.564524
0.231265	-0.04413	-0.66387	0.077429	1.402405	0.853077	-1.76485	-0.11913	-1.15964	-0.36383
-1.22662	0.254026	1.570236	-0.11917	0.451615	0.759458	-1.40271	1.44788	-1.11544	-0.61186
1.160933	1.975503	0.099639	-0.38257	0.722543	-0.96538	-0.52258	1.241161	0.147408	0.454443
0.468949	-1.9692	-2.32085	-0.1513	0.471051	-0.78528	0.819074	0.8982	-1.40831	-0.33261
-1.14654	1.094128	0.161838	0.393951	0.464036	-0.97492	-2.01586	-1.27522	-0.94545	0.601903
-0.05629	-0.77039	-0.65711	0.068733	-0.60208	1.066478	-0.2093	0.058118	0.085823	0.362997
-1.89191	0.459445	-0.83853	-0.56439	0.697433	0.406133	-0.39695	-1.03063	0.501767	-1.02617
-1.13036	1.358444	1.159605	-2.30396	-0.20943	0.102592	-0.50378	1.121782	-2.4168	-1.38602
-1.67741	0.38101	-0.49056	2.244334	-0.09764	-0.67673	-1.04208	0.416868	1.154467	0.818077
1.418717	0.164728	-0.15024	0.056818	1.265943	-0.44833	-1.46963	-1.23639	1.387929	-0.21106
-1.19864	0.094906	0.883167	-0.73596	0.905165	0.751636	-1.03426	1.10383	0.194562	0.205941
-1.1466	-0.08019	-0.48687	-0.94266	0.812426	0.931354	0.564128	0.418738	-0.61717	-0.68532
1.373492	1.864881	0.149167	-1.03886	1.848769	0.76249	0.365523	-0.28257	0.004231	-0.45281
0.024624	-0.15245	1.078065	1.150357	0.130105	0.086684	1.144827	0.096599	-0.94858	0.378201
0.079913	0.18968	-0.68377	-0.29366	-0.68945	-0.56066	-0.05681	0.6886	1.224792	-0.36439
-1.71136	1.179117	0.534943	-0.81923	-0.3276	-0.28863	0.113183	-0.08405	0.373191	0.134423
1.913817	0.017819	-1.21075	0.276654	-0.25798	0.703822	-0.00384	-0.89846	0.49954	1.032614

Descriptive Statistics of Small Samples
N=7

Pair A, V1

Mean	0.450426
Standard E	0.56047
Median	-0.04875
Mode	#N/A
Standard C	1.482863
Sample Va	2.198883
Kurtosis	-0.45948
Skewness	1.030931
Range	3.886953
Minimum	-0.97723
Maximum	2.909723
Sum	3.15298
Count	7
Confidence	1.371421

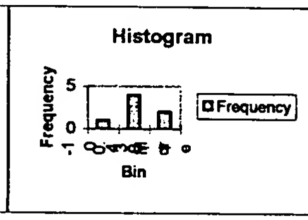
Bin	*Frequency*
-0.97723	1
0.966247	4
More	2



Pair A, V2

Mean	-0.03743
Standard E	0.347565
Median	-0.36238
Mode	#N/A
Standard C	0.919572
Sample Va	0.845612
Kurtosis	1.115894
Skewness	1.169931
Range	2.722531
Minimum	-1.04951
Maximum	1.673026
Sum	-0.26203
Count	7
Confidence	0.850463

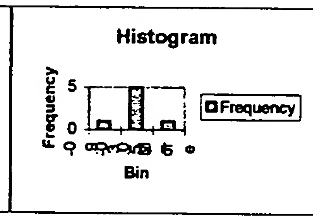
Bin	*Frequency*
-1.04951	1
0.31176	4
More	2



Pair B, V1

Mean	-0.24265
Standard E	0.214852
Median	-0.37308
Mode	#N/A
Standard C	0.568446
Sample Va	0.323131
Kurtosis	2.798664
Skewness	1.340268
Range	1.816522
Minimum	-0.9359
Maximum	0.880622
Sum	-1.69853
Count	7
Confidence	0.525725

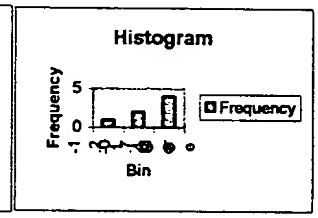
Bin	*Frequency*
-0.9359	1
-0.02764	5
More	1



Pair B, V2

Mean	-0.6785
Standard E	0.14634
Median	-0.42086
Mode	#N/A
Standard C	0.387178
Sample Va	0.149907
Kurtosis	-1.94966
Skewness	-0.6113
Range	0.89182
Minimum	-1.2149
Maximum	-0.32308
Sum	-4.74948
Count	7
Confidence	0.35808

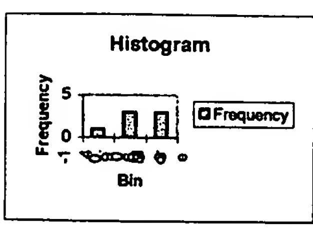
Bin	*Frequency*
-1.2149	1
-0.76899	2
More	4



Pair C, V+B661

Mean	0.027538
Standard E	0.434807
Median	-0.12747
Mode	#N/A
Standard C	1.150391
Sample Va	1.323399
Kurtosis	-1.48493
Skewness	-0.06633
Range	3.058596
Minimum	-1.49615
Maximum	1.562444
Sum	0.192768
Count	7
Confidence	1.063935

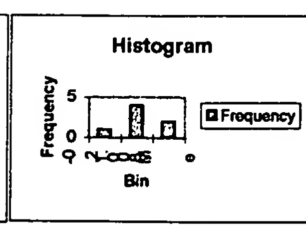
Bin	*Frequency*
-1.49615	1
0.033146	3
More	3



Pair C, V2

Mean	0.784528
Standard E	0.315174
Median	0.634424
Mode	#N/A
Standard C	0.833873
Sample Va	0.695344
Kurtosis	1.668062
Skewness	1.134284
Range	2.571371
Minimum	-0.20891
Maximum	2.362462
Sum	5.491694
Count	7
Confidence	0.771205

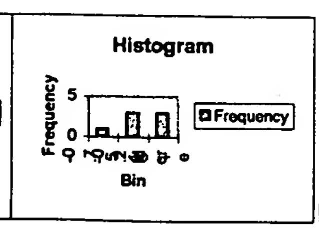
Bin	*Frequency*
-0.20891	1
1.076777	4
More	2



Pair D, V1

Mean	-0.33035
Standard E	0.134676
Median	-0.366
Mode	#N/A
Standard C	0.35632
Sample Va	0.126964
Kurtosis	-1.42221
Skewness	0.403595
Range	0.940578
Minimum	-0.75181
Maximum	0.18877
Sum	-2.31247
Count	7
Confidence	0.329541

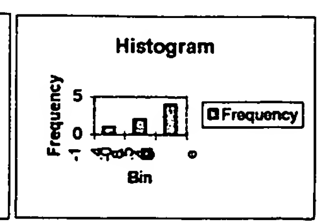
Bin	*Frequency*
-0.75181	1
-0.28152	3
More	3



Pair D, V2

Mean	-0.49912
Standard E	0.276713
Median	-0.5937
Mode	#N/A
Standard C	0.732114
Sample Va	0.535992
Kurtosis	-1.94667
Skewness	-0.16976
Range	1.769019
Minimum	-1.48422
Maximum	0.284799
Sum	-3.49383
Count	7
Confidence	0.677093

Bin	*Frequency*
-1.48422	1
-0.59971	2
More	4

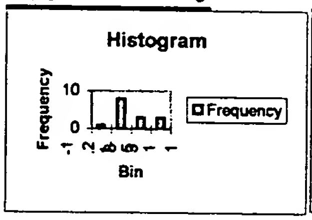


Descriptive Statistics of Small Samples
N=15

Pair A, V1

Mean	0.053905
Standard E	0.337076
Median	-0.20336
Mode	#N/A
Standard [	1.305488
Sample Va	1.704299
Kurtosis	0.269906
Skewness	1.13703
Range	4.193263
Minimum	-1.28354
Maximum	2.909723
Sum	0.808576
Count	15
Confidence	0.722956

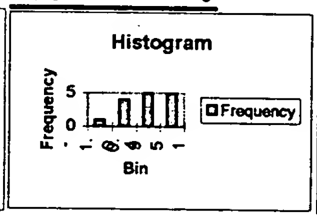
Bin	*Frequency*
-1.28354	1
0.114214	8
1.511969	3
More	3



Pair A, V2

Mean	0.490114
Standard E	0.245419
Median	0.429991
Mode	#N/A
Standard [	0.950502
Sample Va	0.903455
Kurtosis	-1.22001
Skewness	0.091504
Range	3.000946
Minimum	-1.04951
Maximum	1.951441
Sum	7.351706
Count	15
Confidence	0.526371

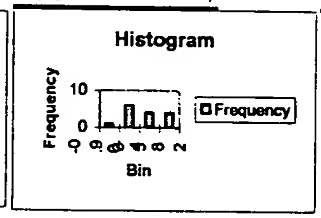
Bin	*Frequency*
-1.04951	1
-0.04919	4
0.951125	5
More	5



Pair B, V1

Mean	0.027154
Standard E	0.161223
Median	-0.03572
Mode	#N/A
Standard [	0.624414
Sample Va	0.389893
Kurtosis	-0.61389
Skewness	0.498485
Range	2.127194
Minimum	-0.9359
Maximum	1.191293
Sum	0.407317
Count	15
Confidence	0.34579

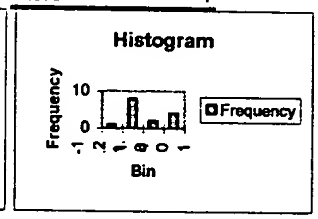
Bin	*Frequency*
-0.9359	1
-0.22684	6
0.482229	4
More	4



Pair B, V2

Mean	0.056505
Standard E	0.299086
Median	-0.36803
Mode	#N/A
Standard [	1.158353
Sample Va	1.341782
Kurtosis	-0.77163
Skewness	0.836512
Range	3.324581
Minimum	-1.2149
Maximum	2.109679
Sum	0.847575
Count	15
Confidence	0.641475

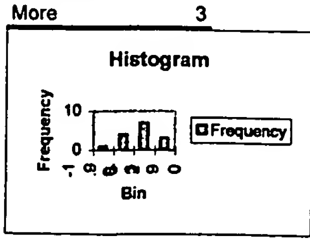
Bin	*Frequency*
-1.2149	1
-0.10671	8
1.001485	2
More	4



Pair C, V1

Mean	-0.3306
Standard E	0.254102
Median	-0.33437
Mode	#N/A
Standard [	0.984133
Sample Va	0.968518
Kurtosis	-0.42573
Skewness	0.290784
Range	3.514931
Minimum	-1.95249
Maximum	1.562444
Sum	-4.95898
Count	15
Confidence	0.544995

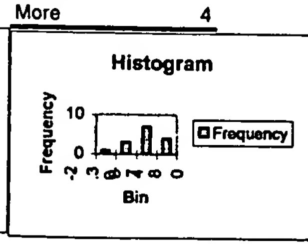
Bin	*Frequency*
-1.95249	1
-0.78084	4
0.390801	7
More	3



Pair C, V2

Mean	0.044406
Standard E	0.335159
Median	0.30914
Mode	#N/A
Standard [	1.298064
Sample Va	1.68497
Kurtosis	-0.33929
Skewness	-0.34152
Range	4.746622
Minimum	-2.38416
Maximum	2.362462
Sum	0.666088
Count	15
Confidence	0.718844

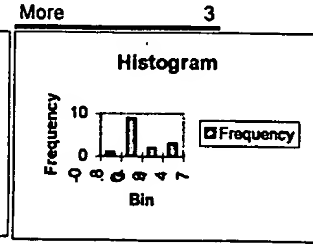
Bin	*Frequency*
-2.38416	1
-0.80195	3
0.780255	7
More	4



Pair D, V1

Mean	0.098899
Standard E	0.211453
Median	-0.1624
Mode	#N/A
Standard [	0.818953
Sample Va	0.670684
Kurtosis	-0.07014
Skewness	0.960032
Range	2.673034
Minimum	-0.83496
Maximum	1.838077
Sum	1.483486
Count	15
Confidence	0.453522

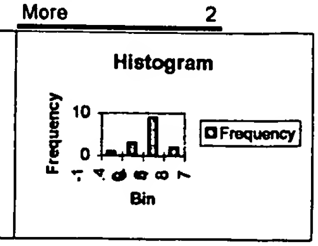
Bin	*Frequency*
-0.83496	1
0.056054	9
0.947065	2
More	3



Pair D, V2

Mean	0.054328
Standard E	0.211194
Median	0.250374
Mode	#N/A
Standard [	0.817952
Sample Va	0.669046
Kurtosis	0.75619
Skewness	-0.17069
Range	3.257642
Minimum	-1.48422
Maximum	1.773422
Sum	0.814913
Count	15
Confidence	0.452967

Bin	*Frequency*
-1.48422	1
-0.39834	3
0.687541	9
More	2

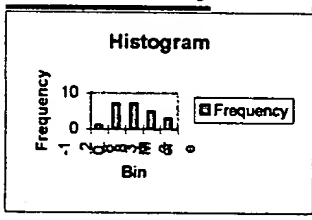


Descriptive Statistics of Small Samples
N=23

Pair A, V1

Mean	0.295621
Standard E	0.238759
Median	0.193411
Mode	#N/A
Standard D	1.145049
Sample Va	1.311138
Kurtosis	-0.3689
Skewness	0.526621
Range	4.193263
Minimum	-1.28354
Maximum	2.909723
Sum	6.799279
Count	23
Confidence	0.495157

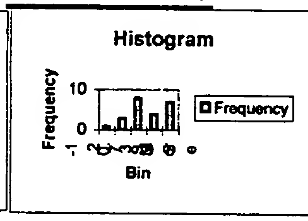
Bin	*Frequency*
-1.28354	1
-0.23522	7
0.813092	7
1.861407	5
More	3



Pair A, V2

Mean	0.433597
Standard E	0.191637
Median	0.325676
Mode	#N/A
Standard D	0.919059
Sample Va	0.844669
Kurtosis	-0.85548
Skewness	0.031131
Range	3.23093
Minimum	-1.27949
Maximum	1.951441
Sum	9.97274
Count	23
Confidence	0.397431

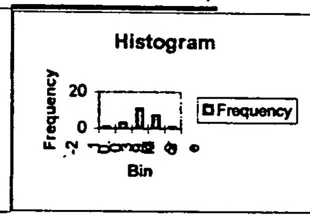
Bin	*Frequency*
-1.27949	1
-0.47176	3
0.335976	8
1.143708	4
More	7



Pair B, V1

Mean	0.057412
Standard E	0.215708
Median	0.029731
Mode	#N/A
Standard D	1.0345
Sample Va	1.07019
Kurtosis	1.374238
Skewness	0.480699
Range	4.902513
Minimum	-2.10911
Maximum	2.793407
Sum	1.320471
Count	23
Confidence	0.447352

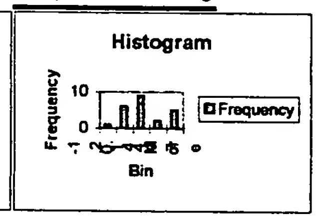
Bin	*Frequency*
-2.10911	1
-0.88348	3
0.34215	11
1.567778	7
More	1



Pair B, V+K1012

Mean	0.145215
Standard E	0.209474
Median	-0.1493
Mode	#N/A
Standard D	1.0046
Sample Va	1.009221
Kurtosis	-0.4539
Skewness	0.729695
Range	3.324581
Minimum	-1.2149
Maximum	2.109679
Sum	3.339934
Count	23
Confidence	0.434422

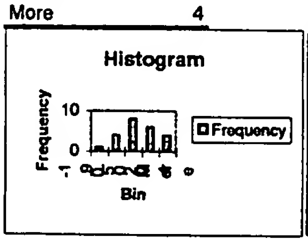
Bin	*Frequency*
-1.2149	1
-0.38376	6
0.447388	9
1.278534	2
More	5



Pair C, V1

Mean	-0.06751
Standard E	0.207896
Median	-0.017
Mode	#N/A
Standard D	0.997033
Sample Va	0.994075
Kurtosis	-0.25868
Skewness	0.19592
Range	3.914167
Minimum	-1.95249
Maximum	1.961681
Sum	-1.55264
Count	23
Confidence	0.43115

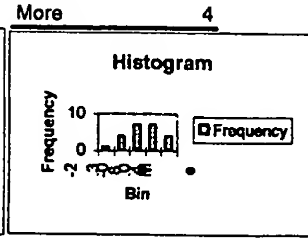
Bin	*Frequency*
-1.95249	1
-0.97394	4
0.004597	8
0.983139	6
More	4



Pair C, V2

Mean	-0.06993
Standard E	0.246672
Median	-0.09116
Mode	#N/A
Standard D	1.182995
Sample Va	1.399478
Kurtosis	-0.3547
Skewness	-0.05823
Range	4.746622
Minimum	-2.38416
Maximum	2.362462
Sum	-1.60834
Count	23
Confidence	0.511566

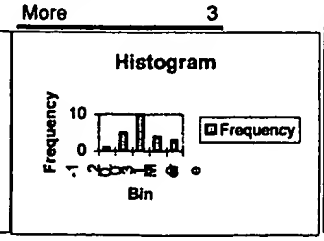
Bin	*Frequency*
-2.38416	1
-1.1975	4
-0.01085	7
1.175807	7
More	4



Pair D, V1

Mean	0.088416
Standard E	0.170752
Median	0.033927
Mode	#N/A
Standard D	0.818899
Sample Va	0.670595
Kurtosis	-0.39191
Skewness	0.513887
Range	3.039275
Minimum	-1.2012
Maximum	1.838077
Sum	2.033562
Count	23
Confidence	0.354119

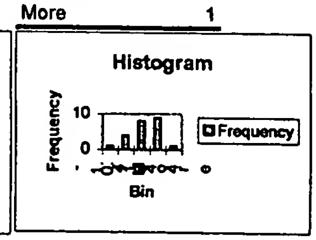
Bin	*Frequency*
-1.2012	1
-0.44138	5
0.318439	10
1.078258	4
More	3



Pair D, V2

Mean	-0.08203
Standard E	0.163013
Median	0.111832
Mode	#N/A
Standard D	0.781785
Sample Va	0.611188
Kurtosis	0.205099
Skewness	0.014878
Range	3.257642
Minimum	-1.48422
Maximum	1.773422
Sum	-1.88678
Count	23
Confidence	0.33807

Bin	*Frequency*
-1.48422	1
-0.66981	4
0.144601	8
0.959011	9
More	1



Sample of Standard N rmal Populati n, Box-Muller Transformation, Sample Count=62

Column1		*Column2*		*Column3*		*Column4*		*Column5*	
Mean	0.096989	Mean	0.196302	Mean	0.104324	Mean	-0.00952	Mean	0.058135
Standard E	0.13961	Standard E	0.122261	Standard E	0.123087	Standard E	0.134324	Standard E	0.127375
Median	0.109633	Median	0.168694	Median	0.103114	Median	-0.1503	Median	0.009687
Mode	#N/A	Mode	#N/A	Mode	#N/A	Mode	#N/A	Mode	#N/A
Standard C	1.099287	Standard C	0.962687	Standard C	0.969186	Standard C	1.05767	Standard C	1.002953
Sample Va	1.208433	Sample Va	0.926765	Sample Va	0.939322	Sample Va	1.118666	Sample Va	1.005915
Kurtosis	-0.54311	Kurtosis	-0.47122	Kurtosis	0.606116	Kurtosis	0.036602	Kurtosis	-0.62364
Skewness	0.270592	Skewness	-0.00894	Skewness	0.104529	Skewness	0.385504	Skewness	-0.04033
Range	4.801632	Range	3.944707	Range	5.114252	Range	4.548292	Range	3.965942
Minimum	-1.89191	Minimum	-1.9692	Minimum	-2.32085	Minimum	-2.30396	Minimum	-2.00426
Maximum	2.909723	Maximum	1.975503	Maximum	2.793407	Maximum	2.244334	Maximum	1.961681
Sum	6.013295	Sum	12.17075	Sum	6.46811	Sum	-0.59005	Sum	3.60437
Count	62	Count	62	Count	62	Count	62	Count	62
Confidence	0.279167	Confidence	0.244477	Confidence	0.246127	Confidence	0.268598	Confidence	0.254702

Sample of Standard Normal Population, Box-Muller Transformation, Sample Count=48

Column1		*Column2*		*Column3*		*Column4*		*Column5*	
Mean	0.029718	Mean	0.25206	Mean	0.107852	Mean	0.017778	Mean	0.116748
Standard E	0.158592	Standard E	0.137522	Standard E	0.145238	Standard E	0.146055	Standard E	0.147548
Median	0.065401	Median	0.190806	Median	0.100486	Median	-0.1503	Median	0.083812
Mode	#N/A	Mode	#N/A	Mode	#N/A	Mode	#N/A	Mode	#N/A
Standard C	1.09876	Standard C	0.952784	Standard C	1.006237	Standard C	1.011897	Standard C	1.022242
Sample Va	1.207273	Sample Va	0.907796	Sample Va	1.012513	Sample Va	1.023936	Sample Va	1.044978
Kurtosis	-0.31892	Kurtosis	-0.26996	Kurtosis	0.722071	Kurtosis	0.254814	Kurtosis	-0.76122
Skewness	0.381712	Skewness	-0.05139	Skewness	0.149183	Skewness	0.4922	Skewness	-0.05751
Range	4.801632	Range	3.944707	Range	5.114252	Range	4.548292	Range	3.914167
Minimum	-1.89191	Minimum	-1.9692	Minimum	-2.32085	Minimum	-2.30396	Minimum	-1.95249
Maximum	2.909723	Maximum	1.975503	Maximum	2.793407	Maximum	2.244334	Maximum	1.961681
Sum	1.426485	Sum	12.09887	Sum	5.176872	Sum	0.853352	Sum	5.603888
Count	48	Count	48	Count	48	Count	48	Count	48
Confidence	0.319046	Confidence	0.276659	Confidence	0.292181	Confidence	0.293824	Confidence	0.296828

Numeric Output of Box-Muller Transformation on Uniform R.V. Sample Sequences

Ten Uniform Sampling Sequences, each separately seeded; made standard normal by Box-Muller Method:

Box-Muller: Standard Normal Random Variable N1 = SQRT(-2*LN(U(Ia))) * COS(2*PI*(U(Ib)))

Box-Muller: Standard Normal Random Variable N2 = SQRT(-2*LN(U(Ia))) * SIN(2*PI*(U(Ib)))

The sets of descriptive statistics, Sampling Count=N represented herein, one for each of ten standard normals is the drawing of Box-Muller standard normal transform variables from Sample Count=500 Sequences The drawings are in sequential order, and the drawings herein, 7<=Sampling Count<=62 are on first 7 drawings of each of ten sequences, then on first 15, first 23, first 48, first 62

Sample f Standard N rmal P pulation, B x-Muller Transf rmation, Sample C unt=62

Column6		Column7		Column8		C lumn9		Column10	
Mean	-0.02677	Mean	-0.2056	Mean	-0.03363	Mean	-0.09963	Mean	-0.03354
Standard E	0.124805	Standard E	0.123796	Standard E	0.097062	Standard E	0.124201	Standard E	0.118063
Median	-0.03285	Median	-0.22023	Median	0.014122	Median	-0.04147	Median	0.033968
Mode	#N/A	Mode	#N/A	Mode	#N/A	Mode	#N/A	Mode	#N/A
Standard D	0.982716	Standard D	0.974768	Standard D	0.764267	Standard D	0.977963	Standard D	0.929633
Sample Va	0.965731	Sample Va	0.950173	Sample Va	0.584104	Sample Va	0.956411	Sample Va	0.864217
Kurtosis	0.254169	Kurtosis	0.386065	Kurtosis	-0.19345	Kurtosis	0.33183	Kurtosis	0.486656
Skewness	0.090306	Skewness	0.468056	Skewness	0.175304	Skewness	-0.15939	Skewness	-0.14884
Range	4.972036	Range	4.840904	Range	3.261751	Range	4.875202	Range	5.053046
Minimum	-2.38416	Minimum	-2.01586	Minimum	-1.48833	Minimum	-2.59152	Minimum	-2.58601
Maximum	2.587876	Maximum	2.825042	Maximum	1.773422	Maximum	2.283682	Maximum	2.467037
Sum	-1.65981	Sum	-12.747	Sum	-2.08523	Sum	-6.1771	Sum	-2.0793
Count	62	Count	62	Count	62	Count	62	Count	62
Confidence	0.249563	Confidence	0.247545	Confidence	0.194088	Confidence	0.248356	Confidence	0.236083

Sample of Standard Normal Population, Box-Muller Transformation, Sample Count=48

Column6		Column7		Column8		Column9		Column10	
Mean	-0.03693	Mean	-0.12866	Mean	0.019805	Mean	-0.10218	Mean	-0.05258
Standard E	0.143687	Standard E	0.148233	Standard E	0.113792	Standard E	0.144669	Standard E	0.126471
Median	-0.00224	Median	-0.21236	Median	0.077359	Median	-0.04147	Median	-0.07049
Mode	#N/A	Mode	#N/A	Mode	#N/A	Mode	#N/A	Mode	#N/A
Standard D	0.995491	Standard D	1.02699	Standard D	0.788373	Standard D	1.002297	Standard D	0.876214
Sample Va	0.991003	Sample Va	1.054709	Sample Va	0.621532	Sample Va	1.004599	Sample Va	0.767751
Kurtosis	-0.23206	Kurtosis	0.200353	Kurtosis	-0.22563	Kurtosis	0.441772	Kurtosis	0.366947
Skewness	-0.14942	Skewness	0.430159	Skewness	0.123458	Skewness	-0.02939	Skewness	0.39663
Range	4.746622	Range	4.840904	Range	3.257642	Range	4.875202	Range	4.111267
Minimum	-2.38416	Minimum	-2.01586	Minimum	-1.48422	Minimum	-2.59152	Minimum	-1.64423
Maximum	2.362462	Maximum	2.825042	Maximum	1.773422	Maximum	2.283682	Maximum	2.467037
Sum	-1.77268	Sum	-6.17575	Sum	0.950643	Sum	-4.90459	Sum	-2.52385
Count	48	Count	48	Count	48	Count	48	Count	48
Confidence	0.28906	Confidence	0.298207	Confidence	0.228919	Confidence	0.291036	Confidence	0.254426

Covariance Matrix of the 10 Standard Normal Random Variables by Box Muller Transformation

N=500

"Column"="Variable"

	Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7	Column 8	Column 9	Column 10
Column 1	0.996043									
Column 2	-0.04105	0.986854								
Column 3	0.031823	0.012184	0.948934							
Column 4	-0.01871	0.01527	0.08836	1.110005						
Column 5	-0.04984	0.017663	0.012141	-0.03094	0.997724					
Column 6	0.071139	-0.05074	0.012547	0.020647	-0.0721	1.113107				
Column 7	0.071779	-0.03356	0.093068	-0.00222	-0.0278	0.006576	0.910356			
Column 8	-0.04405	-0.06172	0.020523	0.06867	-0.06247	0.029204	0.023201	0.866817		
Column 9	-0.02768	-0.09384	0.086771	-0.02058	0.004719	-0.00906	-0.02967	0.041684	0.998309	
Column 10	0.001321	0.036867	-0.02009	0.007213	-0.03914	0.019259	0.000287	-0.05617	0.028016	1.101594

Numeric Output of Box-Muller Transf rmati n n Uniform R.V. Sample Sequences

Standard N rmal Random Variable = SQRT(-2*LN(U(Ia))) * COS(2*PI*(U(Ib)))

Ui, Ui+1	Ui+1, Ui	Ui, Ui+2	Ui+2,Ui+1	Ui, Ui+3	Ui+1,Ui+4	Ui+3,Ui+6	Ui+2,Ui+4	Ui+2,Ui+1	Ui, Ui+4
1.118872	-1.58307	-0.3654	-0.69476	-1.6074	-0.17458	-0.1134	-0.63083	-0.60704	-0.19364
-1.76097	1.095513	-0.65562	-0.38205	1.169318	-0.82681	1.436748	-1.02918	-0.86182	0.460256
0.819046	-1.70328	0.689477	-0.02854	1.889882	1.961846	-0.84363	0.177596	-2.05045	-1.34441
0.345203	0.204496	-0.92404	-0.77499	-1.04786	0.013833	0.113221	0.904402	0.019001	-1.14238
0.478426	-0.13827	-0.31349	-0.53047	2.227761	-1.26497	-0.42605	1.722225	1.101179	-0.63685
0.290076	-0.04331	-0.92676	0.363484	-0.31316	0.073279	1.691046	2.1972	-0.47974	-0.57526
-2.43136	1.315783	0.087435	0.838746	-0.91111	-0.13445	-0.85647	-2.59147	1.471318	0.540155
0.875077	-0.08907	0.928218	-0.26028	0.237688	-0.85193	-0.25699	-1.71653	0.757634	1.255192
0.34939	2.550201	1.687155	0.738857	0.277501	-1.43518	0.667979	1.259439	-0.81323	1.451371
0.062037	1.660399	2.094043	0.492714	-0.30331	-0.24448	0.373244	-0.39485	-0.21708	-0.39549
1.609129	0.175943	-0.04757	0.356864	0.603472	-1.36693	-0.55363	0.062117	-0.33635	-0.1031
2.434943	-0.326	-2.02535	0.199056	0.494561	0.289172	-1.51283	-1.49272	0.772803	2.864968
1.345628	-0.48641	0.260463	0.173991	-0.02524	-0.00521	-0.82695	-1.50974	-0.39952	-1.74264
0.360795	-1.02509	0.295266	-1.62831	1.807545	0.025402	-1.06673	0.153552	-0.78919	-0.22163
1.731175	-0.28156	0.275341	0.007381	-0.24168	-0.11768	-0.97916	-0.081	0.163717	-0.71647
-0.62307	0.313387	1.404234	1.103462	1.624226	-1.65122	0.329539	0.71382	0.768283	-0.70069
-0.87475	-0.07465	-0.6308	-1.23152	-0.01054	0.303147	-0.50781	0.436575	-0.13266	-0.07592
-1.38044	0.431862	-0.68003	-2.97003	0.092794	0.772726	0.090116	0.232528	1.198599	0.507581
-0.67922	0.34882	0.408809	2.001131	-0.16524	0.968062	0.854391	0.087578	-0.50793	0.155114
-0.99403	0.442876	-1.05106	0.854716	0.706641	-0.63139	0.435125	0.041212	0.411069	0.740367
-0.86632	1.412082	0.647699	-0.13145	-0.71215	0.179567	-1.55996	-0.92316	0.734622	0.769818
1.215633	1.473143	0.799967	0.353926	0.873707	0.315572	-1.80975	0.161371	-0.92429	1.63163
-0.42369	1.695935	0.968578	0.635108	1.888079	-1.08472	-0.50511	-1.40106	-1.63588	-1.0055
-1.10512	0.338555	-0.2391	0.55001	-1.25599	-0.16656	-0.62467	0.107881	0.887989	0.199775
-0.43394	-0.05453	1.517265	1.447604	-1.90638	0.329465	0.128868	-1.06969	-0.87143	0.144489
1.525457	2.934186	-0.37609	0.336223	0.384899	-0.6235	0.651922	-0.25848	-1.53285	0.422612
0.077303	0.863244	-2.18056	-0.78451	-0.99535	-1.21938	0.616698	-1.11639	-0.73814	-0.10068
0.654517	-0.57947	1.363245	-0.09364	0.810267	0.407644	1.602652	-0.79756	0.430047	0.531883
-0.00522	-0.52859	-0.0172	2.493412	-0.76081	-0.66871	-0.37544	0.470082	0.332	-0.13278
1.574736	-1.79791	1.634293	1.766889	0.208081	1.003906	0.106538	1.207018	0.96122	-0.13165
0.297123	0.234462	-0.42488	0.320057	-0.6848	1.776916	0.713178	-0.63947	-0.51942	-0.74219
-0.67472	0.298095	0.506237	-0.20057	-0.05348	0.71191	-1.45559	-1.61208	0.039996	-1.96631
0.339364	-1.4859	0.255537	0.509092	-0.76652	1.512484	-1.99612	-0.44984	0.72621	0.254585
0.627231	0.088969	-0.06105	-0.1547	-1.53484	1.136838	1.257833	0.946974	-1.25988	-1.59282
-0.23088	1.184502	0.595199	0.683635	-0.90104	0.816371	1.202563	-1.28733	-0.44968	-0.92467
-0.38325	-2.0636	1.437681	0.292331	-0.82502	-0.27798	-2.29716	-1.93788	1.127317	0.240214
0.002002	1.473674	0.338102	-1.82452	-1.18774	0.517247	-0.21324	-0.36935	-0.66443	0.788009
-0.45133	0.842243	0.772931	0.458165	-0.16006	-0.56454	-1.08125	-0.51276	-0.36472	-0.96747
0.23212	-0.61883	-0.34457	-1.01465	1.064423	-0.21212	0.670957	-0.79752	1.041947	1.089883
0.636441	-0.08057	0.643937	-0.51585	0.316166	2.098419	-0.46843	-1.15965	-2.76228	1.738551
0.72588	-1.18912	0.910637	0.713088	-0.66988	1.002968	0.375314	-0.87682	1.403065	0.429474
1.654785	-0.61133	2.570961	-0.27269	-0.2332	0.524699	1.216853	0.796632	-0.94486	-0.69531
-1.47551	-0.15168	-2.28509	-0.4528	-0.66813	0.308534	-0.69861	1.115909	-0.19814	0.020134
-1.42791	1.155833	-0.14551	0.682247	0.885726	1.747474	0.108283	2.130862	0.894232	-0.20483
-1.19652	-1.13817	-1.01927	-0.32955	1.17313	-1.55026	1.146678	-0.03867	-0.25785	1.937894
-0.81113	-2.58546	-0.25469	0.287016	-0.56946	-1.76813	-0.34083	2.357887	0.835689	0.899246
1.398032	1.686079	0.053655	0.731198	1.682325	-0.74918	-0.03518	-0.00698	-1.06282	0.329185
0.163039	0.109541	-1.15186	-0.53875	0.004967	-0.99481	-0.79499	-0.14999	-0.37061	1.225057

Section III

Financial Engineering Calculator

Departure

Over the last decade, the operations within the financial community have grown in complexity and scope. Ten years ago, it was not required that MBA financial candidates master first-year calculus. Now, to be a "financial engineer", this knowledge is but a fundamental prerequisite. See *Exhibit 1: FE Curriculum.*

Yet, regardless of these new challenges, every financial professional, whether a financial analyst, actuary, risk or credit manager, bond, stock, mortgage or insurance professional bears a pocket calculator. It is surprising to those not in these fields how the calculator is used, and is on hand, daily in these professions.

For both the student and professional alike, the calculator is almost always nearby, even at the most senior level or in the most computerized information technology environment. I can testify to both counts, having degrees and awards in several financial disciplines, spread amidst eight years in banking and industry.

In the Fall of 1995, I took prerequisite mathematics, Calculus II and Linear Algebra, at a local college, classmates ranging from pre-teen to middle age. The teacher, who taught high school calculus, was an exponent of the Texas Instruments-80 series calculator, encouraging every student to own one and use it.

This teacher was enthusiastic about the TI-82 and -85, but disciplined at test time. Then he required the memory to be cleared; for polar coordinate work, he impounded these calculators. This inconvenience to the student is avoided by a two-hour disable feature on memory and/or graphics, or teacher's storage device.

Later that semester, I upgraded from a mid-Eighties Casio scientific calculator to a TI-85, finding it entertaining to program multiple variant algorithms for graphical display, then pressing "graph", to observe them being sketched out as artwork. Short coded demos on-board memory can engage interest in functions.

Another general improvement to the TI-80 series, and the newer TI-90 series, in addition to those above, is to create a reference compendium, an abbreviated dictionary and encyclopedia of mathematical, scientific and engineering terms, theorems, equations and thinkers. The graphical displays afford this output.

Notwithstanding but alternately, such a resource could be software, as RAM or ROM or accessible through the interface as afforded by the various current TI models. I have yet to exhaust my TI-85's RAM, nor do I have much use for the LINK; exchanges of variables, memory and programs with other units is rare.

As pertains to any of the TI series 80 and 90 calculators, these are remarkably versatile devices, but the user is required to develop and code the variables, programs and equations which are typically required for their field of interest. This hurdle often keeps the users from taking a fuller advantage of the unit's ability.

This raises the cause for value-added software that can be packaged as targeted sets for different functional specialties. When I matriculated to Polytechnic, I relied exclusively upon the TI-85 for calculation that first semester, since I did not have a computer then, fulfilling all assignments, while posting a 3.7 GPA.

I was forced by my circumstances to code equations and to explore its gamut of features. I showed my classmates its potential and within a month or two, almost everyone had bought one. However, they learned the pain and tedium of this skilled labor, with everyone wanting my memory, or just giving up on it.

After all it is "just" a calculator, meaning there is already too much needing done with one's time.

Therefore, within or outside the sufficiency of available memory, it is evident that value-added encoded software is attractive to the marketplace, further enhancing and facilitating the usage of TI series calculators in academic and professional spheres. Various suggestions are cited, *Appendix A: Enhancement.*

Financial Engineering Calculator

General Market f r a Financial Calculator

While attending the mathematics courses in Fall 1995, I purchased my TI-85 then with its purpose being to serve as my calculator during the coming full-time financial engineering program. In 1987, when I attended the NYU Stern School of Business, I bought a Casio scientific calculator with statistical features.

Casio was very useful, it even modeled a complex financial derivative, but was outdated by units in 1995. These new scientific and engineering calculators, starting at $90, now had an equation solver feature plus simultaneous equation, more quantitative, analytic and math functions, graphing and multi-row displays.

Back in the 1980's the HP 12C "RPC" was used frequently in the financial industries. Also, niche business calculators, little more than a 1970's base calculator with hard-key elementary financial functions for mortgage, leasing, lending, bond and annuity computations, existed, with both categories still sold today.

I did not purchase an HP 12C in 1987 because it provided little of use beyond elementary functions, not having related features about these functions: its comes lacking a clock, time zone, calendar; it is without industry rate conversions or the full day-count and accrued interest conventions, etc.; and is weaker in math.

To be frank, most of the available business calculators then and now are simple, inflexible gizmos limited to basic calculations made by the car, bond or insurance salesman, bookkeeper or mortgage agent. This math is simple routines, being typically all that these calculators can do at their functional math ability.

I purchased my TI-85 in December 1995, for $100, reasonably aware of the market offering for my financial calculation needs. A TI-80's series, or comparable HP 38G ($100), were logical for their equation solver, accepting one must code the functions, etc., storing these to the unit's memory, plus a graph display.

The selection of available financial calculators (but not, "financial engineering"), i.e. the TI BAII ($40), which has a solver feature, other TI units, the HP 12C ($80) or the HP 17BII ($100), was ruled out - for the cost of the TI-85 ($100-120), I could perform most every finance equation, combination or process.

I thought I would have a computer to perform my intensive calculations, spread sheets and numerical results, but found myself armed only with my TI-85 for the first semester. Using my equations and notes from memory in TI-85's solver, simult, statistical and display features, I performed all work perfectly.

Had I chosen any other calculator, save the HP 19BII, its top financial calculator based on a TI-85-comparable HP 38G ($99) graphing calculator, I could not have performed my assignments entirely or at all. Still, with this, the HP 19BII ($130) or other unit, almost all equations and data must be coded into memory.

Today, the situation remains that the ready-to-use financial engineering calculator does not exist. The TI-86, even with its "Financial Functions", simply mirrors the HP 19BII, though without a clock or calendar, currency and unit conversion, IRDA port, but with bigger display. Code your own is still required.

A review of my equations and data which I coded into my TI-85 during my 1½ years of full-time financial engineering studies is noted in *Appendix B: RAM Contents.* Contrast these FE functions to those of the "financial calculators", still I have not taxed its 28KB RAM. *See Exhibit 2: FE Calculator Products.*

With regard to my memory utilization in the TI-85, I do have few data sets or programs stored. I do however, also have a number of reference and resource items, listed and contained in RAM. Together, these equations and reference items just 10 KB of RAM, making full compendia and algorithm sets realistic.

General Market f r a Financial Calculat r, cont'd.

Using a TI-86, $130, one gains triple the RAM (96K) for equations, data and programs. My items in RAM memory do not include programs that could be coded to run simulations, equations or processing in combinations. Thus, most certainly, the TI-86 offers vital RAM for this code and data for only $30 more.

Of particular note is the TI-83 ($130), adding classes of features invaluable to an FE calculator (absent on the TI-85; but on the TI-86) - traceable scatterplots and histograms, stored column formulas, inferential statistics, probability distributions and logistic and sine regressions. The TI-86 is a valid FE base.

Working from the large TI-92 platform, 3D graphing is merged with larger display, but only 68K RAM is available for discretionary RAM, and still no ROM facility. At a price of $350, more than triple a TI-85, the question becomes about putting 3D graphing into a pocket-sized version. Both are valued in FE.

The TI-92 Plus ($80) is added memory and coding, raising the base 92 several steps. This software module doubles the RAM to 188K (remember, the TI-86 has 96K for $130). The Plus module expands mathematics, graphing and functionality, as well as adding user-available ROM of 384K, for a total of $430.

The HP 32SII "RPC" programmable scientific calculator sets a price mark ($65) by delivering the valuable equation solver feature, but has a very low small memory (0.4 KB), a single-line display and sub-TI-85 mathematical features. For 3D graphing, memory and advanced calculus, spend over six times more.

All financial calculators have statistical features limited to only two variables. Most do not have a clock, time zone, or calendar for current pricing default; few have industry rate conversions or the full day-count and accrued interest conventions. Random number and probability distribution are poor (expt. TI-83).

In Fall 1996, during my second FE semester, I was fortunate to purchase my TI TravelMate 6030 at Texas Instrument's generosity, not just because subsequently the Pentium I desktop market collapsed, but because the academic needs had made a computer essential. I switched into MS Excel for heavy calculation.

Along with Excel ($300, $500 MSOffice), I have a MatLab suite ($1,000+), incl. its financial, statistical and optimization toolboxes and its ExcelLink. These software packages, and FinCad ($400), with their classes of functions and qualities are summarized in *Exhibit 3: Professional FE Calculation Products*.

Now, there is a very important class of features which are absent in whole from current calculators: the export and import of data sets, via coaxial, serial, parallel, TCP/IP or comcard interface, in ASCII format, or MSExcel, MatLab, or C. The TI-Graph Link ($40), only facilitates storage and accesses features.

Either building interface capability into the FE calculator, or having an interface device, is of prime importance and value. This keeps the FE calculator at the junction of hardware and software, boosting its usefulness The device is not a CBL($250) or CBR ($130); these lack standard interfaces and file conversion.

Regard inventor's RAM Contents, *Appendix B*; notice that its rich smorgasbord of functional equations and reference items for the financial engineer - its mathematicians, traders, analysts, managers, actuaries and students - can surpass or complement that of its notably more expensive professional brethren.

These software packages require a desktop computer ($1,000+, +OS), as palmtops ($600) can not run them. Nonetheless, they have their advanced features such as spreadsheets, table and chart generation, and quick statistical processing. To run sequences, simulations and system processes usually requires coding.

Appendix C: Functions and Features of Preferred FE Calculator, functionally specifies a FE device.

Financial Engineering Calculator

EXHIBITS

Exhibit 1: Financial Engineering (FE) Curriculum

M. Sc. Financial Engineering. Capital Markets. Polytechnic University. Brooklyn, NY.

Center of Finance & Technology, Department of Management, two year, 48 credit, M. Sc. FE program. First program of its kind, designed and approved by the International Association of Financial Engineers.

Courses Prerequisites: mathematics, economics, programming or science/engineering

financial accounting	financial services	corporate policy	c programming
fixed-income analytics	fixed-income markets	futures/options/swaps	probability
data systems design	continuous time finance	portfolio management	applied statistics
numerical analysis	financial engineering	advanced fixed-income	adv. derivatives.

M. Sc. Financial Mathematics. University of Chicago. Chicago.

Department of Mathematics, University of Chicago, one year program, in three consecutive quarters. Designed to produce specialization in pricing models for financial derivatives. "Program consists of four components: mathematics, probability theory, economics, financial applications and simulations."

Courses/Topics Prerequisites: undergraduate in mathematics, finance or science/engineering

numerical methods	o.d.e.	capm theory	diffusion equation
eigenvalue problems	p.d.e.	portfolio theory	signal processing
neural networks	model identification	discrete models	continuous time
black-scholes/diffusion	martingales	stochastic calculus	utility theory.

M. Sc. Risk Management. The College of Insurance. New York City.

Graduate Division, two year, 48 credit program, in addition to its MBA, DRI and actuarial degrees. M. Sc. program provides insurance management with modern financial, actuarial and risk sciences.

Courses Prerequisites: undergraduate in mathematics, economics, finance, insurance

managerial economics	quantitative analysis	financial management	risk & insurance
financial risk management	insurance company mgnt.	alternative risk finance	risk management
applied statistics	numerical analysis	operations research	credibility theory
contingencies	survival models	reinsurance	loss distributions

M. Sc. Computational Finance. Carnagie Mellon. Pittsburgh and New York City.

Center for Financial Analysis and Securities Trading (FAST), five seven-week mini-semesters, each with four courses. Designed for those in quantitative finance; committed to integrate the technology and analytics of today's global financial markets. Its centerpiece is the modern FAST Trading Room.

Courses/Topics Prerequisites: mathematics, programming, finance or science/engineering

real analysis	probability	stochastic calculus	p.d.e.
financial analysis	price theory	investment analysis	options/futures
fixed-income valuation	statistics	time series analysis	simulation
object programming	algorithms	software engineering	monte carlo.

Exhibit 2: Financial Engineering Calculator Products

comparison of existent market offering for financial engineering calculators:

Unit	Price	Memory/Display	Features	FE Functions
		Texas Instruments:		
TI Graphing Calculators:				
TI-85	$99-130	28K/ 128x64, 8x21	graphing, programs, trig., equation solver, simult statistics, diff. eq. graphing phys. constants, conversions	none coded
TI-83	$120	27K/ 96x64, 8x16	TI-85: + table, list, column, trace plots, hists.; split screen; inferent. stats., prob. distr., assembly lang. programming multi graph styles, no diff. eq.	=TI-86 (ROM-coded)
TI-86	$130	128K; 96K user/ 128x64, 8x21	all TI-85 and TI-83: no spl.sc., large lists, user defined functions euler, init. cond., field graphing	cashflow, date amort., TVM (via web)
TI-92	$350	68K user/ 240x128, varies	all TI-86: + spl.sc., 3D surface symbolic manip., interact. geom., no: diff eq. graph., eigen., phys.c.	none coded
TI-92 Plus	+$80	188K user/348K ROM 240x128, varies	all TI-86 and TI-92: + 3D spin, contour plot, symb. units and ode	none coded
TI Scientific Calculators:				
TI 68		K limited, single line	simult; no stat, solver, store, clock	none coded
TI Business Calculators:				
TI BA II Plus	$40	K limited, not known single line display stat;	solver; non-graph; modest math; no simult, store, clock, calndr	TVM, cashflow amort,2day-count
TI BA Real Estate		K limited, single line	= TI BA II Plus, no stat, less math	piti, tvm, arm
		Hewlett Packard:		
HP Business Calculators:				
HP 12C	$80-90	K limited single line	no solver, lists, edit, store, sort non-graph, "120" functions "reverse polish notation"	loans, lease, amort. (TVM) irr, cashflow; no conversns no clock, calendar
HP 19BII	$130	K not known; 4x23 LCD	similar to TI-86 graphing, solver, stat "450" business, stat, science, math IRDA port to HP printer (optional)	all HP fin functions currency, conversions clock, calendar
HP Scientific Calculators:				
HP 32SII	$65	0.4K, single line	"100" science, math functions; solver, reverse polish notation, store	none coded

Exhibit 3: Professional Financial Engineering C mputation Products

Note: None of the TI LINK, CBL or CBR or the HP interface (IRDA) options provide for data (feed) signal capture, for exchange of formatted data with other software platforms, or provide for integration of calculator with computer. These serious shortcomings prevent current calculators from supporting full FE preferred functionality and features. Consequently, the financial engineer must rely on computer-based, professional grade financial software packages. FE calculator units provided with hardware/software interface for data capture/exchange can compete in this market.

Professional grade financial engineering software packages:

	MS Excel	**MatLab**	**FinancialCAD**
Description	MS Office spreadsheet, charts statistical, finance add-ins	3D matrix-based numeric program scientific/math suite, Excel-Link	Windows-based module: VB, Visual C++, Excel
Requirements	MS OS, Pentium computer	MS OS, Pentium computer	MS OS, Pentium or Alpha
Price	$400 MS Excel ($600 as Office) plus: MS OS Pentium (≥$1000)	$500 platform, to $5000 as suite plus: MS OS Pentium (≥$1000)	$400 (+Excel), $2500 devl plus: OS, computer, VB,C
Features	MS computations, moderate math spreadsheets, charts, tables, data two variable descriptive statistics easy export to MS Word, Office	robust numeric, graphical package requires learning MatLab coding suites provide topical algorithms simulation, C, system ID, AI, etc.	finance application library and workbook page results as module on Excel frame or VB or C development
FE Functions	ROM-coded financial functions date, clock, calendar functions strong, but basic, fixed-income cashflow, irr, amort, tvm, etc. no derivatives, options, models spreadsheet affords limited lattice statistical add-in is easy to use simulation possible using VB MS Excel is business data format Excel used by third-party devel.	coded in Financial Toolbox ($400) "95 functions in seven categories" requires Statistics + Optimization Toolboxes (add per desk, $1500+) tech. charting financial data, ex/im clock, calendar, full conversions fixed-income security valuation portfolio analysis and optimization basic derivatives, Black-Scholes old binomial model, Simulink (+$)	comprehensive valuation: fixed-income securities; fixed-income derivatives; money-market securities; equities, equ. derivatives; interest rate derivatives; currency, fx derivatives; commodities, derivatives; basic, B-S, option models; exotic options and models

Hardware performing computations from financial software:

While small, Personal Organizers ($80-$400) do not support financial software. Palmtops, smaller than a TI-92, such as by Psion, Sony and HP, run MS Windows CE OS, support limited Excel but not the packages above. However, CE OS will improve. Hardware packages, i.e. the top HP 620LX ($850) has color VGA, 640x240, 80-column, display, direct printing, 16 MB ROM and 16 MB RAM, a 75-Mhz processor, Type II PCMCIA card slot and flash.

Laptops, from Mini ($1300) to Alpha Dec (to $10000), support full MS OS and the financial software packages above, as can Desktop Workstations ($700-$15000) with Intel 486, Pentium I, II and Pro chipsets or Alpha Dec processors. Top systems, elite workstations, servers with RAID storage, mini- to main-frame computers run financial enterprise class software, specialized institutional platforms for consolidated data, research, risk, trade management.

Financial enterprise management s ftware platf rms and programs:

Numerous packages exist, from JP Morgan's 4:15 VaR ($25K p.a.) or RiskMetrics, to Algorithmic's Risk Series and Infinity/Sungard ($100K-$M). These have full data API, premier coded applications and scaleable system integration.

APPENDICES

Appendix A: General Enhancement of Texas Instruments (80, 90 Series) Calculators

Two-hour disable feature on memory and/or graphics, internal or as outside device, for tests

this enables the TI calculators to be used in **test environments** without further ado or loss
presently for the student who does not have a computer to download into, **memory** is lost
disable memory and/or graphics functions for a time period, so the calculator can be used:
an **internal** disable feature with timed duration, using the processor's clock to count time
or teacher's **storage**, disabling device, 30KB-96(188)KB per calculator (student) per test.

Short coded demos in on-board memory of interesting usage, topics, subjects and formulae

it's always **fun** for an electronic device to have simple programs, showcasing capabilities
it's always good for dedicated devices to stimulate interest and **learning** in their subject
the depth of **features** available in the TI calculators often remain hidden from casual use:
demos on topics, functions and formulas in memory not the Guidebook or ASM program:
the "Guidebook's" require **coding**; the ASM are not so organized, conceived or focused
for **examples** use the "Guidebook", or my visual art graphic generations, sample, "Insect":
graph polar: $r1 = 5\cos(2\Pi)$; $r2 = 2+2\cos(2\theta)$; $r3 = 5-2\tan(5\theta)$; $r4 = 4+4\sin(2+2\theta)$ et al.
add brief elaboration and context to educate, to inform; see also examples in Appendix B.

create resource compendia, RAM/ROM sets, providing coded functions and items on-board

not much on-board memory need be taken up by **assorted** demos, being fixed-coded items
by **executing** demos on user command, stored graphics, results or images are not required
add required list, group or function for the variety of **subject** expositions ala encyclopedia:
target assemblage of reference compendia to varied educational levels of math and science
high school version supports **teaching** of geometry, algebra, probability, calculus, sciences
elementary/college versions help educate; **scaleable** to lower TI units, and 80's to 92 Plus
make advanced **specialized** resources per industry, as modules loaded to RAM or installed
per electrical, mechanical, environmental, financial engineering; math, physics, astronomy
such **items** include today's methods, theorems, formulae, procedures, pre-coded functions
compendia add pivotal resources: references, equations, algorithms, processes, programs.

professional standard industry-specific software, pre-loaded or accessible through interface

develop the reference resources along with subject functions coded to existent calculators
arm the presently empty TI memories with compendium of equations, conversions, etc.
consider some to full pre-loading; modules of other fields are downloaded LINK to RAM

value-added software can be packaged as desirable assets for different operational specialties

TI has application archives, esp. for TI-85, though /calc-apps/85 is not very user-friendly
proper subject **archives** must be arranged, supplementing existent with newly coded items
the **new** archives would be value-added property to integrate, sell, install, or avail over net
this **software** can be pre-loaded (opt. delete), be sold separately, or on-line at charge/free
assess additional access and memory capacity, i.e. RAM/ROM cards, ext/int drive/storage
technology paths calculator units towards improved digital interfaces, bus, PCMCIA, disk.

Appendix B: Synopsis f RAM Contents in Financial Engineering Calculator

Inventor's Financial Equations and References coded for use in TI-85

Equations

AI CorpB	AI TB	Annuity	Bond Equiv Yield	Bin 1, Bin2, Bin 3
Binomial	BS	Bond	BonK, BonV	Brown
CBT	CLT	Comp	Con, Conadj, Condp	Convexity
DeltaP, dP	dPdY	DurMod	DurMc	DV01
FFOTD	Forward	FX	Hedge, HR	MDS
Min1,Min2,Min3	Mortgage	MPC	Muni	OAS3 (example)
OCF	PAY, PAY1	PR, PRBond	PRCalB, PRMunat	PRO
PROMOD	PROPC	PTIC	PV	SPC
Spot	Swap	FXSwap	Tbill1, TB2,	TBT
TDCap	V	Var	W	BoxMuller

Reference Items

Bernoulli	Black	Borel-Cantelli	Boundary	Brownian
B-S	BSC	CAPM	Cheby	Correlation
CoVar	Credit	cut-off	distfunc	EN
EQU	EX	Floater	FOCF	GenFunc
GcS	HeathJar	Inde	Intre	Ito
Lambda	lease	martingale	minrisk	mpr
partition	PCP	Poisson	Port	RandomW
replication	riskadverse	SPC	strong	theorfut
tokens	tree	utility	weak	weight

Published Sources for Financial Matter, Equations and References

Books

Benninga, Simon. Numerical Techniques in Finance. MIT Press. ISBN 0-262-52141-5.
Bowers, et al. Actuarial Mathematics. Society of Actuaries. ISBN 0-938959-46-8.
Fabozzi, Frank. Bond Markets, Analysis and Strategies. Prentice Hall. ISBN 0-13-339151-5.
Fabozzi and Fabozzi. Municipal Bond Portfolio Management. Irwin. ISBN 1-55623-672-7.
Gastineau, Gary. Dictionary of Financial Risk Management. Probus. ISBN 1-55738-443-6.
Grimmett and Welsh. Probability an Introduction. Oxford University. ISBN 0-19-853264-4.
Herzog, Thomas. Introduction to Credibility Theory. Actex. ISBN 1-56698-221-9.
Hogg and Klugman. Loss Distributions. Wiley. ISBN 0-471-87929-0.
Hull, John. Options, Futures, and other Derivative Securities. Prentice Hall. ISBN 0-13-639014-5.
Jarrow and Turnbull. Derivative Securities. South-Western. ISBN 0-538-86271-8.
Merton, Robert. Continuous-Time Finance. Blackwell. ISBN 0-631-18508-9.
Peters, Edgar. Chaos and Order in the Capital Markets. Wiley. ISBN 0-471-13938-6.
Strang and Nguyen. Wavelets and Filter Banks. Wellesley. ISBN 0-9614088-7-1.
Tu, Pierre. Dynamical Systems. Springer-Verlag. ISBN 3-540-57661-4.
Tuckman, Bruce. Fixed Income Securities. Wiley. ISBN 0-471-11214-3.
Weibull, Jurgen. Evolutionary Game Theory. MIT Press. ISBN 0-262-23181-6.

Periodicals

Derivatives Strategy. Derivatives Strategy and Tactics. New York.
IEEE Spectrum. The Institute of Electrical and Electronics Engineers, Inc. New York.
The Journal of Financial Engineering. International Association of Financial Engineers. New York.
The Journal of Fixed Income. Institutional Investor, Inc. New York.

Appendix C: Functi nal Specificati n f Financial Engineering Calculator

Note: all pricing are target approximations of unit retail without applying charge for FE functions/software.

I. Feasible under present prior art hardware technology:

on hardware hardwire, ROM, RAM or LINK as **equations, variables, programs, reference items, data**:

clock, date, calendar, default value present time/date
intervals between dates, coupons, valuation, exercise, expiration
day-count conventions, instrument standards, conversions
fixed-income general valuations (annuity, mortgage, lease, bond, rates and yields)
fixed-income advanced valuations (variable cash-flows, inverse, MBS, sinking, optionality)
fixed-income derivative valuations (options, futures basis, hedge ratios, swaps, FX dP/dY)
fixed-income and derivative sensitivities (duration, convexity, delta, gamma, theta, dtheta)
fixed-income yield curve building (spot, risk-free short rates and forward curves)
accounting standards, (GAAP, statutory, derivatives, credit quality, risk-adjusted capital)
financial statement and performance ratios, operating ratios of financial criterion
credit and ratings grade conventions, calculating ratings and spread approximations
insurance ratios, pricing, quantitative methods
reinsurance forms and pricing of excess of loss, facultative, treaty varieties
actuarial mathematics and sciences, loss distributions, contingencies, survival models
standard normal and lognormal random number generation, selectable N, descriptive statistics of sample
simulations by lattice, brownian motion, random sequence generation, interpolation
portfolio management of VaR, performance analytic measures
direct approximations by derivation, linear algebra, symbolic, integration, interpolation
mapping to charts, display multiple list and graphical display (to 3D)
one, two and more variable statistics and multi-factor regression
time series and artificial intelligence data mining, normalization procedures
inferential and descriptive statistics, probability distributions
real-time and formatted data loading and serial, IRDA and TCP/IP
stored column formulas, spreadsheet capability, data set manipulation
split screen, display size pixels 128x64, 8x21 display characters
trace, overlay (or by split screen) and combine scatterplots, historgrams, interpolations, results

II. New designs and/or upward migration:

for pocket-size: non-PCMCIA, with **data translator interfacing** with external formats (≅ \$200);
could be built off TI-85 series specifications: adding:
formatted data loading/conversion via advanced LINK/software or translator interface/ IRDA port.

for **enhanced processing capabilities**: attempted within pocket-size dimensions (≤\$600)
memory to 8 MB (++)
60 Mhz or higher processor
serial/parallel or IRDA to external drives, storage, modem, ethernet or to interface device
near real-time communications and database processing bussing.

within pocket or TI-92 size, **full palmtop hardware**: incl. PCMCIA, port, drive, storage, OS (≅\$1000)
real-time data signal via serial, parallel, TCP/IP, IRDA and/or PCMCIA
mobile and network communication via PCMCIA modem, ethernet or port
as TI-92 size: on-board storage, hard drive, PCMCIA slot(s)
as pocket size: has multiple peripheral interfaces or outboard multiple interface device.